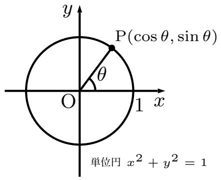
三角関数公式連関表

三角関数の定義，相互関係



$$\begin{cases}\sin\theta = (\text{P の } y \text{ 座標})\\ \cos\theta = (\text{P の } x \text{ 座標})\\ \tan\theta = \dfrac{\sin\theta}{\cos\theta} = (\text{OP の傾き})\end{cases}$$

$$\cos^2\theta + \sin^2\theta = 1 \quad \xrightarrow[\div \cos^2\theta]{} \quad 1+\tan^2\theta = \frac{1}{\cos^2\theta}$$

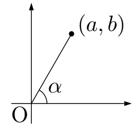
還元公式

$$\begin{cases}\sin(-\theta) = -\sin\theta\\ \cos(-\theta) = \cos\theta\\ \tan(-\theta) = -\tan\theta\end{cases} \quad \begin{cases}\sin(\theta+2n\pi) = \sin\theta\\ \cos(\theta+2n\pi) = \cos\theta\\ \tan(\theta+2n\pi) = \tan\theta\end{cases} \quad \begin{cases}\sin(\theta+\pi) = -\sin\theta\\ \cos(\theta+\pi) = -\cos\theta\\ \tan(\theta+\pi) = \tan\theta\end{cases}$$

$$\begin{cases}\sin\left(\theta+\dfrac{\pi}{2}\right) = \cos\theta\\ \cos\left(\theta+\dfrac{\pi}{2}\right) = -\sin\theta\\ \tan\left(\theta+\dfrac{\pi}{2}\right) = -\dfrac{1}{\tan\theta}\end{cases} \quad \begin{cases}\sin\left(\dfrac{\pi}{2}-\theta\right) = \cos\theta\\ \cos\left(\dfrac{\pi}{2}-\theta\right) = \sin\theta\\ \tan\left(\dfrac{\pi}{2}-\theta\right) = \dfrac{1}{\tan\theta}\end{cases} \quad \begin{cases}\sin(\pi-\theta) = \sin\theta\\ \cos(\pi-\theta) = -\cos\theta\\ \tan(\pi-\theta) = -\tan\theta\end{cases}$$

合成公式 (sin への合成)

$$a\sin\theta + b\cos\theta$$
$$= \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right)$$
$$= \sqrt{a^2+b^2}(\cos\alpha\sin\theta + \sin\alpha\cos\theta)$$
$$= \sqrt{a^2+b^2}\sin(\theta+\alpha)$$



加法定理（出発点！）

$$\begin{cases}\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{cases}$$

$$\begin{cases}\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta\\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta\end{cases}$$

$$\begin{cases}\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\\ \tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\end{cases}$$

2 倍角の公式

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= 1 - 2\sin^2\alpha \quad \text{(sin 表示)}$$
$$= 2\cos^2\alpha - 1 \quad \text{(cos 表示)}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

3 倍角の公式

$$\begin{aligned}\sin 3\alpha &= \sin(\alpha+2\alpha)\\ &= \sin\alpha\cos 2\alpha + \cos\alpha\sin 2\alpha\\ &= \sin\alpha(1-2\sin^2\alpha) + \cos\alpha\cdot 2\cos\alpha\sin\alpha\\ &= \sin\alpha - 2\sin^3\alpha + 2(1-\sin^2\alpha)\sin\alpha\\ &= 3\sin\alpha - 4\sin^3\alpha\end{aligned}$$

$$\begin{aligned}\cos 3\alpha &= \cos(\alpha+2\alpha)\\ &= \cos\alpha\cos 2\alpha - \sin\alpha\sin 2\alpha\\ &= \cos\alpha(2\cos^2\alpha - 1) - \sin\alpha\cdot 2\sin\alpha\cos\alpha\\ &= 2\cos^3\alpha - \cos\alpha - 2(1-\cos^2\alpha)\cos\alpha\\ &= 4\cos^3\alpha - 3\cos\alpha\end{aligned}$$

半角の公式

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2} \iff \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \iff \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\tan^2\frac{\alpha}{2} = \frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}} = \frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

和積 (積和) 変換公式　$(\alpha+\beta=A,\quad \alpha-\beta=B)$

$$\sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin\alpha\cos\beta \quad \begin{cases}\sin\alpha\cos\beta = \dfrac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}\\ \sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}\end{cases}$$

$$\sin(\alpha+\beta)-\sin(\alpha-\beta) = 2\cos\alpha\sin\beta \quad \begin{cases}\cos\alpha\sin\beta = \dfrac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}\\ \sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}\end{cases}$$

$$\cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos\alpha\cos\beta \quad \begin{cases}\cos\alpha\cos\beta = \dfrac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}\\ \cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}\end{cases}$$

$$\cos(\alpha+\beta)-\cos(\alpha-\beta) = -2\sin\alpha\sin\beta \quad \begin{cases}\sin\alpha\sin\beta = -\dfrac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}\\ \cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}\end{cases}$$

$\sin\alpha$, $\cos\alpha$, $\tan\alpha$ の $t\left(=\tan\dfrac{\alpha}{2}\right)$ による有理関数表示 (※数 III の三角関数の積分等で使用)

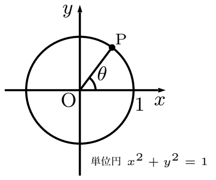
$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}} = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2t}{1+t^2}$$

$$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}} = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-t^2}{1+t^2}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2t}{1-t^2}$$

三角関数公式連関表（問題 ✍：20 分）

三角関数の定義，相互関係



$$\begin{cases} \sin\theta = \\ \cos\theta = \\ \tan\theta = \end{cases}$$

$\longrightarrow$

還元公式

$$\begin{cases} \sin(-\theta) = \\ \cos(-\theta) = \\ \tan(-\theta) = \end{cases} \qquad \begin{cases} \sin(\theta+2n\pi) = \\ \cos(\theta+2n\pi) = \\ \tan(\theta+2n\pi) = \end{cases} \qquad \begin{cases} \sin(\theta+\pi) = \\ \cos(\theta+\pi) = \\ \tan(\theta+\pi) = \end{cases}$$

$$\begin{cases} \sin\left(\theta+\dfrac{\pi}{2}\right) = \\ \cos\left(\theta+\dfrac{\pi}{2}\right) = \\ \tan\left(\theta+\dfrac{\pi}{2}\right) = \end{cases} \qquad \begin{cases} \sin\left(\dfrac{\pi}{2}-\theta\right) = \\ \cos\left(\dfrac{\pi}{2}-\theta\right) = \\ \tan\left(\dfrac{\pi}{2}-\theta\right) = \end{cases} \qquad \begin{cases} \sin(\pi-\theta) = \\ \cos(\pi-\theta) = \\ \tan(\pi-\theta) = \end{cases}$$

合成公式

$a\sin\theta + b\cos\theta$

$=$

加法定理（出発点！）

$$\begin{cases} \sin(\alpha+\beta) = \\ \sin(\alpha-\beta) = \end{cases}$$

$$\begin{cases} \cos(\alpha+\beta) = \\ \cos(\alpha-\beta) = \end{cases}$$

$$\begin{cases} \tan(\alpha+\beta) = \\ \tan(\alpha-\beta) = \end{cases}$$

2 倍角の公式

$\sin 2\alpha =$

$\cos 2\alpha =$

$=$ (sin 表示)

$=$ (cos 表示)

$\tan 2\alpha =$

3 倍角の公式

$\sin 3\alpha =$

$\cos 3\alpha =$

半角の公式

$\sin^2\alpha = \quad \Longleftrightarrow \quad \sin^2\dfrac{\alpha}{2} =$

$\tan^2\dfrac{\alpha}{2} =$

$\cos^2\alpha = \quad \Longleftrightarrow \quad \cos^2\dfrac{\alpha}{2} =$

和積 (積和) 変換公式　$(\alpha+\beta=A,\quad \alpha-\beta=B)$

$$\begin{cases} \sin(\alpha+\beta)+\sin(\alpha-\beta) = \\ \sin(\alpha+\beta)-\sin(\alpha-\beta) = \end{cases}$$

$\begin{cases} \sin\alpha\cos\beta = \\ \sin A + \sin B = \end{cases}$

$\begin{cases} \cos\alpha\sin\beta = \\ \sin A - \sin B = \end{cases}$

$$\begin{cases} \cos(\alpha+\beta)+\cos(\alpha-\beta) = \\ \cos(\alpha+\beta)-\cos(\alpha-\beta) = \end{cases}$$

$\begin{cases} \cos\alpha\cos\beta = \\ \cos A + \cos B = \end{cases}$

$\begin{cases} \sin\alpha\sin\beta = \\ \cos A - \cos B = \end{cases}$

$\sin\alpha$，$\cos\alpha$，$\tan\alpha$の $t\left(=\tan\dfrac{\alpha}{2}\right)$ による有理関数表示 (※数 III の三角関数の積分等で使用)

$\sin\alpha =$

$\cos\alpha =$

$\tan\alpha =$