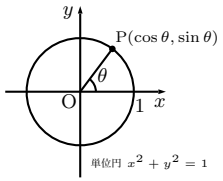
三角関数公式連関表

三角関数の定義，相互関係



$$\begin{cases}\sin\theta = (\text{P の } y \text{ 座標})\\ \cos\theta = (\text{P の } x \text{ 座標})\\ \tan\theta = \dfrac{\sin\theta}{\cos\theta} = (\text{OP の傾き})\end{cases}$$

$$\cos^2\theta+\sin^2\theta=1 \xrightarrow[\div\cos^2\theta]{} 1+\tan^2\theta=\frac{1}{\cos^2\theta}$$

還元公式

$$\begin{cases}\sin(-\theta)=-\sin\theta\\ \cos(-\theta)=\cos\theta\\ \tan(-\theta)=-\tan\theta\end{cases}\quad\begin{cases}\sin(\theta+2n\pi)=\sin\theta\\ \cos(\theta+2n\pi)=\cos\theta\\ \tan(\theta+2n\pi)=\tan\theta\end{cases}\quad\begin{cases}\sin(\theta+\pi)=-\sin\theta\\ \cos(\theta+\pi)=-\cos\theta\\ \tan(\theta+\pi)=\tan\theta\end{cases}$$

$$\begin{cases}\sin\left(\theta+\dfrac{\pi}{2}\right)=\cos\theta\\ \cos\left(\theta+\dfrac{\pi}{2}\right)=-\sin\theta\\ \tan\left(\theta+\dfrac{\pi}{2}\right)=-\dfrac{1}{\tan\theta}\end{cases}\quad\begin{cases}\sin\left(\dfrac{\pi}{2}-\theta\right)=\cos\theta\\ \cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta\\ \tan\left(\dfrac{\pi}{2}-\theta\right)=\dfrac{1}{\tan\theta}\end{cases}\quad\begin{cases}\sin(\pi-\theta)=\sin\theta\\ \cos(\pi-\theta)=-\cos\theta\\ \tan(\pi-\theta)=-\tan\theta\end{cases}$$

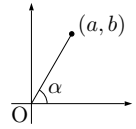
合成公式 (sin への合成)

$$a\sin\theta+b\cos\theta$$

$$=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\sin\theta+\frac{b}{\sqrt{a^2+b^2}}\cos\theta\right)$$

$$=\sqrt{a^2+b^2}(\cos\alpha\sin\theta+\sin\alpha\cos\theta)$$

$$=\sqrt{a^2+b^2}\sin(\theta+\alpha)$$



加法定理（出発点！）

$$\begin{cases}\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta\\ \qquad\text{シンコスコスシン！}\\ \sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta\end{cases}$$

$$\begin{cases}\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta\\ \qquad\text{コスコスシンシン！}\\ \cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta\end{cases}$$

$$\begin{cases}\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\\ \quad 1\text{干}(ほす)\text{タンタン タン上}(と)\text{タン！}\\ \tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\end{cases}$$

2 倍角の公式

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$=1-2\sin^2\alpha \quad (\sin\text{ 表示})$$

$$=2\cos^2\alpha-1 \quad (\cos\text{ 表示})$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

3 倍角の公式

$$\sin 3\alpha=\sin(\alpha+2\alpha)$$
$$=\sin\alpha\cos 2\alpha+\cos\alpha\sin 2\alpha$$
$$=\sin\alpha(1-2\sin^2\alpha)+\cos\alpha\cdot 2\cos\alpha\sin\alpha$$
$$=\sin\alpha-2\sin^3\alpha+2(1-\sin^2\alpha)\sin\alpha$$
$$=3\sin\alpha-4\sin^3\alpha \quad (\text{サンシャイン引いて司祭が参上す})$$

$$\cos 3\alpha=\cos(\alpha+2\alpha)$$
$$=\cos\alpha\cos 2\alpha-\sin\alpha\sin 2\alpha$$
$$=\cos\alpha(2\cos^2\alpha-1)-\sin\alpha\cdot 2\sin\alpha\cos\alpha$$
$$=2\cos^3\alpha-\cos\alpha-2(1-\cos^2\alpha)\cos\alpha$$
$$=4\cos^3\alpha-3\cos\alpha \quad (\text{良い子のみんなで引っ張る神輿}(みこし))$$

半角の公式

$$\sin^2\alpha=\frac{1-\cos 2\alpha}{2}\iff\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}\iff\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$$

$$\tan^2\frac{\alpha}{2}=\frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}=\frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

和積 (積和) 変換公式　$(\alpha+\beta=A,\quad\alpha-\beta=B)$

$$\begin{cases}\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta\\ \sin(\alpha+\beta)-\sin(\alpha-\beta)=2\cos\alpha\sin\beta\\ \cos(\alpha+\beta)+\cos(\alpha-\beta)=2\cos\alpha\cos\beta\\ \cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta\end{cases}\quad\begin{cases}\sin\alpha\cos\beta=\dfrac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\}\\ \sin A+\sin B=2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}\\ \cos\alpha\sin\beta=\dfrac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}\\ \sin A-\sin B=2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}\\ \cos\alpha\cos\beta=\dfrac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\}\\ \cos A+\cos B=2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}\\ \sin\alpha\sin\beta=-\dfrac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}\\ \cos A-\cos B=-2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}\end{cases}$$

$\sin\alpha$, $\cos\alpha$, $\tan\alpha$ の $t\left(=\tan\dfrac{\alpha}{2}\right)$ による有理関数表示 (※数 III の三角関数の積分等で使用)

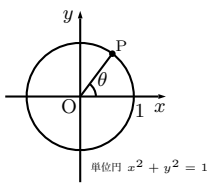
$$\sin\alpha=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$
$$=\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$$
$$=\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$=\frac{2t}{1+t^2}$$

$$\cos\alpha=\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}$$
$$=\frac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}}$$
$$=\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$$
$$=\frac{1-t^2}{1+t^2}$$

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$
$$=\frac{2t}{1-t^2}$$

三角関数公式連関表（問題 ✍：20 分）

三角関数の定義，相互関係



$$\begin{cases}\sin\theta = \\ \cos\theta = \\ \tan\theta = \end{cases}$$

$\longrightarrow$

還元公式

$$\begin{cases}\sin(-\theta) = \\ \cos(-\theta) = \\ \tan(-\theta) = \end{cases} \qquad \begin{cases}\sin(\theta+2n\pi) = \\ \cos(\theta+2n\pi) = \\ \tan(\theta+2n\pi) = \end{cases} \qquad \begin{cases}\sin(\theta+\pi) = \\ \cos(\theta+\pi) = \\ \tan(\theta+\pi) = \end{cases}$$

$$\begin{cases}\sin\left(\theta+\dfrac{\pi}{2}\right) = \\ \cos\left(\theta+\dfrac{\pi}{2}\right) = \\ \tan\left(\theta+\dfrac{\pi}{2}\right) = \end{cases} \qquad \begin{cases}\sin\left(\dfrac{\pi}{2}-\theta\right) = \\ \cos\left(\dfrac{\pi}{2}-\theta\right) = \\ \tan\left(\dfrac{\pi}{2}-\theta\right) = \end{cases} \qquad \begin{cases}\sin(\pi-\theta) = \\ \cos(\pi-\theta) = \\ \tan(\pi-\theta) = \end{cases}$$

合成公式

$a\sin\theta + b\cos\theta$

$=$

加法定理（出発点！）

$$\begin{cases}\sin(\alpha+\beta) = \\ \sin(\alpha-\beta) = \end{cases} \qquad \begin{cases}\cos(\alpha+\beta) = \\ \cos(\alpha-\beta) = \end{cases}$$

$$\begin{cases}\tan(\alpha+\beta) = \\ \tan(\alpha-\beta) = \end{cases}$$

2 倍角の公式

$\sin 2\alpha =$

$\cos 2\alpha =$

$=$ (sin 表示)

$=$ (cos 表示)

$\tan 2\alpha =$

3 倍角の公式

$\sin 3\alpha =$ $\qquad\qquad$ $\cos 3\alpha =$

半角の公式

$\sin^2\alpha = \iff \sin^2\dfrac{\alpha}{2} =$

$\cos^2\alpha = \iff \cos^2\dfrac{\alpha}{2} =$

$\tan^2\dfrac{\alpha}{2} =$

和積 (積和) 変換公式 $(\alpha+\beta=A,\ \alpha-\beta=B)$

$$\begin{cases}\sin(\alpha+\beta)+\sin(\alpha-\beta) = \\ \sin(\alpha+\beta)-\sin(\alpha-\beta) = \end{cases} \qquad \begin{cases}\sin\alpha\cos\beta = \\ \sin A+\sin B = \end{cases} \qquad \begin{cases}\cos\alpha\sin\beta = \\ \sin A-\sin B = \end{cases}$$

$$\begin{cases}\cos(\alpha+\beta)+\cos(\alpha-\beta) = \\ \cos(\alpha+\beta)-\cos(\alpha-\beta) = \end{cases} \qquad \begin{cases}\cos\alpha\cos\beta = \\ \cos A+\cos B = \end{cases} \qquad \begin{cases}\sin\alpha\sin\beta = \\ \cos A-\cos B = \end{cases}$$

$\sin\alpha,\ \cos\alpha,\ \tan\alpha$ の $t\left(=\tan\dfrac{\alpha}{2}\right)$ による有理関数表示 (※数 Ⅲ の三角関数の積分等で使用)

$\sin\alpha =$ $\qquad$ $\cos\alpha =$ $\qquad$ $\tan\alpha =$