2025 慶應(薬)

[1]

(1)

(i) $f(x)=\left(x-\frac{a}{2}\right)^2-\frac{a^2}{4}+a+2$

$\downarrow$ $\forall x\in\mathbb{R},\ f(x)\geqq 0$

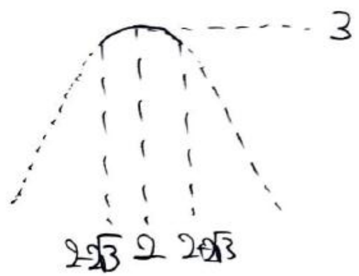
$-\frac{a^2}{4}+a+2\geqq 0$

$\iff a^2-4a-8\leqq 0$

$\iff \underline{2-2\sqrt{3}\leqq a\leqq 2+2\sqrt{3}}$ //

(ii)

$$M=\begin{cases} f(-2) & (a\leqq -4)\\ -\frac{a^2}{4}+a+2 & (-4\leqq a\leqq 6)\\ f(3) & (a\geqq 6)\end{cases}$$

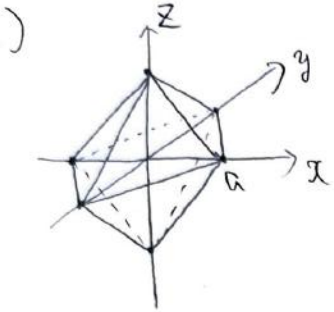


$a=\underline{2}$ // で $\max M=\underline{3}$ //

$f(x)=(x-1)^2+3$

$M=f(-2)=\underline{12}$ //

(2)



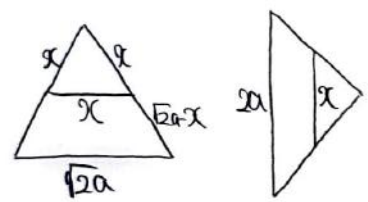
(i) (Sの体積)

$=2a\cdot 2a\cdot\frac{1}{2}\cdot a\cdot\frac{1}{3}\cdot 2$

$=\underline{\frac{4}{3}a^3}$ //

(ii)

(正方形の辺) $=x$



$\downarrow$

$\frac{x}{\sqrt{2}a}\cdot a=\frac{2a-x}{2a}\cdot a$

$\iff x=\frac{2a-x}{\sqrt{2}}$

$\iff (\sqrt{2}+1)x=2a$

$\iff x=2(\sqrt{2}-1)a$

(Uの体積) $=\{2(\sqrt{2}-1)a\}^3$

$=\underline{(40\sqrt{2}-56)a^3}$ //

(3)

$f(x)=\left|\frac{1}{-x\cdot 10^{-x}\cdot\log 10}\right|$

$=\frac{1}{\log 10}\cdot\frac{1}{|x|}10^x$

$=\frac{1}{\log 10}\cdot\frac{10^x}{x}\quad(x>0)$

$f'(x)=\frac{1}{\log 10}\cdot\frac{(\log 10)10^x\cdot x-10^x}{x^2}$

$=\frac{10^x\{(\log 10)x-1\}}{(\log 10)x^2}$

x	0	$\cdots$	$\frac{1}{\log 10}$	$\cdots$
$f'(x)$		$-$		$+$
$f(x)$		$\searrow$		$\nearrow$

$x=\frac{1}{\log 10}=\frac{1}{0.69+1.61}\fallingdotseq\underline{0.43}$ //

最小値

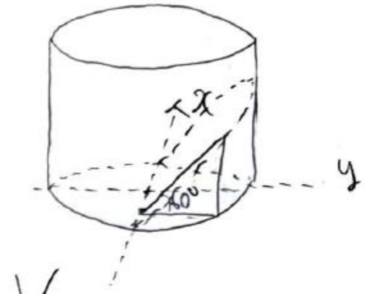
$f\left(\frac{1}{\log 10}\right)=10^{\frac{1}{\log 10}}$

$=10^{\log_{10}e}$

$=e$

$\fallingdotseq\underline{2.72}$ //

(4)



V

$=\int_{-2}^{2}\sqrt{4-x^2}\cdot\sqrt{3}\sqrt{4-x^2}\cdot\frac{1}{2}\,dx$

$=\sqrt{3}\int_0^2(4-x^2)\,dx$

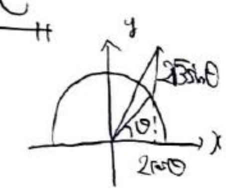
$=\underline{\frac{16\sqrt{3}}{3}}$ //

S_1

$=\int_{-2}^{2}2\sqrt{4-x^2}\,dx$

$=2\cdot 2\pi=\underline{4\pi}$ //

S_2



$=\int_0^{\pi}2\sqrt{3}\sin\theta\cdot\frac{1}{2}\cdot 2^2\,d\theta$

$=4\sqrt{3}\int_0^{\pi}\sin\theta\,d\theta$

$=\underline{8\sqrt{3}}$ //

(5)

$z=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n}$

$z^k=\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$

(i)

$S_n=$  $\times n$

$=\frac{1}{2}\sin\frac{2\pi}{n}\times n$

$=\underline{\frac{n}{2}\sin\frac{2\pi}{n}}$ //

$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}}\cdot\pi=\underline{\pi}$ //

(ii)

$\sum_{k=1}^{n-1}z^k$

$=\frac{1-z^n\ 1}{1-z}$

$=\underline{0}$ //

(iii)

$\alpha=\frac{z+z^2+z^4}{3}$

$\beta=\frac{z^3+z^5+z^6}{3}$

↓

$\alpha+\beta=\frac{1}{3}\cdot\frac{z-z^7}{1-z}=-\frac{1}{3}$

$\alpha\beta=\frac{z^4+z^5+z^7+z^5+z^7+z+z^7+z^2+z^3}{9}$

$=\frac{2}{9}$

α・βは$x^2+\frac{1}{3}x-\frac{2}{9}=0$の解.

$\underline{\alpha=\frac{-1+\sqrt{7}i}{6}}$ // $\underline{\beta=\frac{-1-\sqrt{7}i}{6}}$ //

[II]

(1) $H_0: p=0.4$

$H_1: p>0.4$

Rは$N\left(p,\frac{p(1-p)}{n}\right)$

則) $\underline{N(0.4,0.00024)}$ //

(2)

$Z=\frac{\frac{n}{1000}-0.4}{\sqrt{0.00024}}$は標準正規分布

に従うので,有意水準5%の

片側検定であると棄却する

ためには

$\frac{\frac{n}{1000}-0.4}{\sqrt{0.00024}}\geqq1.645$

$\Leftrightarrow \frac{n-400}{\sqrt{240}}\geqq1.645$

$\Leftrightarrow n\geqq1.645\times4\sqrt{15}+400$

$=1.645\times4\times1.7\times2.2+400$

$=424.6092$

$\underline{425}$ //

[III]

$y=\frac{1}{3}x+\sqrt{\frac{1}{9}x^2+8}$ $(y\geqq2\sqrt{2})$

$y-\frac{1}{3}x=\sqrt{\frac{1}{9}x^2+8}$

↓ 2乗

$y^2-\frac{2}{3}xy=8$

$\Leftrightarrow y^2-8=\frac{2}{3}xy$

$\Leftrightarrow x=\frac{3}{2y}(y^2-8)$

↓ 逆関数

$\underline{y=\frac{3}{2}\left(x-\frac{8}{x}\right)}$ // $\underline{(x\geqq2\sqrt{2})}$ //

(2)

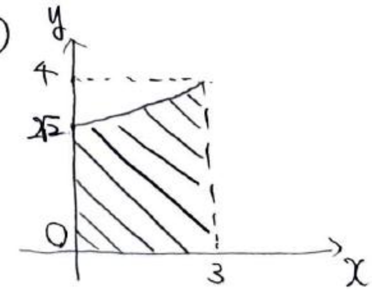
$\int_{2\sqrt{2}}^{4}\frac{3}{2}\left(x-\frac{8}{x}\right)dx$

$=\frac{3}{2}\left[\frac{1}{2}x^2-8\log|x|\right]_{2\sqrt{2}}^{4}$

$=\frac{3}{2}\left(8-8\log4-4+8\log2\sqrt{2}\right)$

$=\underline{6-6\log2}$ //

(3)



$\int_0^3 f(x)\,dx$

$=12-\int_{2\sqrt{2}}^{4}g(x)\,dx$

$=12-(6-6\log2)$

$=\underline{6+6\log2}$ //

[IV]

(1) P(5回目までに当たり出る)

$= 1 - P$(5回目まですべて外れ)

$= 1 - \left(\frac{2}{3}\right)^5 = \underline{\frac{211}{243}}$ //

(2)

P(n回目に当たり)

$= \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$

E_n

$= 50n \cdot \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$

$= \frac{50n}{3}\left(\frac{2}{3}\right)^{n-1} = 25n\left(\frac{2}{3}\right)^n$

$\frac{E_n}{E_{n+1}}$

$= \frac{25n\left(\frac{2}{3}\right)^n}{25(n+1)\left(\frac{2}{3}\right)^{n+1}}$

$= \frac{3n}{2(n+1)} = \frac{10}{7}$

$\Leftrightarrow 21n = 20n + 20$

$\therefore \underline{n = 20}$ //

(3)

$\lim_{n\to\infty} \frac{E_n}{E_{n+1}} = \lim_{n\to\infty} \frac{3}{2\left(1+\frac{1}{n}\right)} = \underline{\frac{3}{2}}$ //

(4)

$S_n = \sum_{k=1}^{n} E_k$ とおく.

$S_n = 25 \cdot \frac{2}{3} + 50\left(\frac{2}{3}\right)^2 + \cdots + 25n\left(\frac{2}{3}\right)^n$

$\frac{2}{3}S_n = \quad 25\left(\frac{2}{3}\right)^2 + \cdots + 25(n-1)\left(\frac{2}{3}\right)^n + 25n\left(\frac{2}{3}\right)^{n+1}$

$-)$

$\frac{1}{3}S_n = 25 \cdot \frac{2}{3} + 25\left(\frac{2}{3}\right)^2 + \cdots + 25\left(\frac{2}{3}\right)^n - 25n\left(\frac{2}{3}\right)^{n+1}$

⋮

$S_n = \underline{150 - 50(n+3)\left(\frac{2}{3}\right)^n}$ //

$\lim_{n\to\infty} S_n = \underline{150}$ //