2022 東邦大(医)

1

焦点 $\sqrt{104^2-40^2}$

$=\sqrt{144\cdot 64}$

$=96$

$\therefore (96,0),(-96,0)$

(焦点までの距離の和)

$=2\cdot 104=\underline{208}$

2

連立して

$3x+4k+3=-2kx-3$

$\Leftrightarrow (2k+3)x=-4k-6$

$\Leftrightarrow (2k+3)(x+2)=0$

$k=-\frac{3}{2}$のとき一致する

$x=-2$のとき

$4+4k-3=0$

$\Leftrightarrow k=-\frac{1}{4}=\underline{\frac{-1}{4}}$

共有点 $\underline{(-2,0)}$

3

$\int_0^1 x\sqrt{1-x}\,dx$ $\quad 1-x=t \Leftrightarrow 1-t=x$

$=\int_1^0 (1-t)\sqrt{t}(-1)dt$

$=\int_0^1 (t^{\frac{1}{2}}-t^{\frac{3}{2}})dt$

$=\left[\frac{2}{3}t^{\frac{3}{2}}-\frac{2}{5}t^{\frac{5}{2}}\right]_0^1$

$=\frac{2}{3}-\frac{2}{5}=\underline{\frac{4}{15}}$

$f(x)=x\sqrt{1-x}+x^2\times a$

$\downarrow$

$a=\int_0^1 (t\sqrt{1-t}+at^2)dt$

$=\frac{4}{15}+\frac{a}{3} \quad \therefore a=\underline{\frac{2}{5}}$

4

⑪⑫○○○

$P_1=\dfrac{1+{}_3C_2+1\cdot 3}{{}_5C_2}$

$=\underline{\frac{7}{10}}$

n回目 $\quad$ n+1回目

$P_n \xrightarrow{\frac{7}{10}} P_{n+1}$

$1-P_n \nearrow \longleftarrow \frac{1\cdot 2}{{}_5C_2}=\frac{1}{5}$

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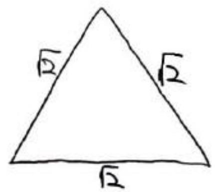
$P_{n+1}=\frac{7}{10}P_n+\frac{1}{5}(1-P_n)$

$=\underline{\frac{1}{2}P_n+\frac{1}{5}}$

5

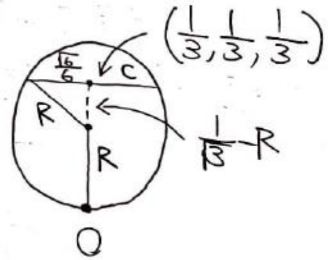


$S=\frac{r}{2}(a+b+c)$

$\frac{1}{2}\cdot\sqrt{2}\cdot\sqrt{2}\sin 60^\circ=\frac{r}{2}\cdot 3\sqrt{2}$

$\sqrt{3}=r\cdot 3\sqrt{2}$

$\therefore r=\frac{1}{\sqrt{6}}=\underline{\frac{\sqrt{6}}{6}}$



三平方

$R^2=\frac{1}{6}+\left(\frac{1}{\sqrt{3}}-R\right)^2$

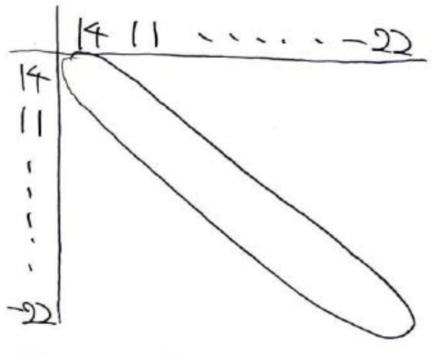
$\Leftrightarrow 0=\frac{1}{2}-\frac{2}{\sqrt{3}}R$

$\therefore R=\underline{\frac{\sqrt{3}}{4}}$

6

$\underset{a_1}{14},\ \underset{a_2}{11},\ \cdots\ \underset{a_{13}}{-22}$

${}_{13}C_2=13\cdot 6=\underline{78}$



(78コの積の和)

$=\left[(14+11+\cdots+(-22))^2-\sum_{k=1}^{13}(-3k+17)^2\right]\times\frac{1}{2}$

$=\left\{\left[(14-22)\times 13\times\frac{1}{2}\right]^2-\sum_{k=1}^{13}(9k^2-102k+17^2)\right\}\times\frac{1}{2}$

$=\left\{16\cdot 13^2-\frac{9}{6}\cdot 13\cdot 14\cdot 27+102\cdot\frac{1}{2}\cdot 13\cdot 14-17^2\cdot 13\right\}\times\frac{1}{2}$

$= \{16\cdot 13 - 21\cdot 27 + 51\cdot 14 - 17^2\} \times \frac{13}{2}$

$= \underline{429}$ //

7

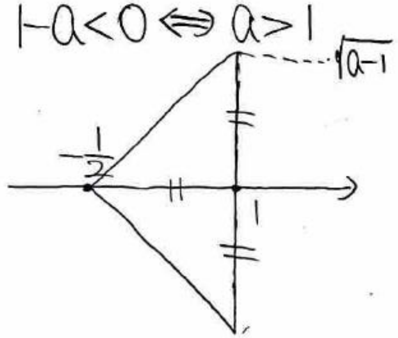
$2x^3 - 3x^2 + 2(a-1)x + a = 0$

$(2x+1)(x^2 - 2x + a) = 0$

$\therefore x = -\frac{1}{2},\ 1 \pm \sqrt{1-a}$

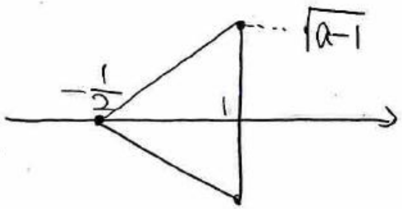
必要条件

$1 - a < 0 \iff a > 1$



$\sqrt{a-1} = \frac{3}{2}$

$a - 1 = \frac{9}{4} \quad \therefore \underline{a = \frac{13}{4}}$ //

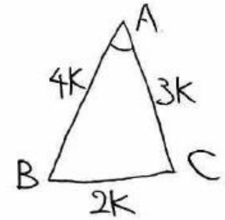


$\sqrt{a-1} = \frac{\sqrt{3}}{2}$

$a - 1 = \frac{3}{4} \quad \therefore \underline{a = \frac{7}{4}}$ //

8

$a : b : c = 2 : 3 : 4$



(余) $\cos A = \frac{16+9-4}{2\cdot 4\cdot 3} = \underline{\frac{7}{8}}$ //

$\sin A = \frac{\sqrt{15}}{8}$

(正) $R = \frac{a}{2\sin A} = \frac{4\cdot 2k}{\sqrt{15}}$

$\frac{1}{2}\cdot 4k\cdot 3k\cdot\frac{\sqrt{15}}{8} = \frac{r}{2}\cdot 9k$

$\iff \frac{3\sqrt{15}}{2}k = 9r$

$\iff r = \frac{\sqrt{15}k}{6}$

$\therefore \frac{r}{R} = \frac{\sqrt{15}}{8k} \times \frac{\sqrt{15}k}{6} = \underline{\frac{5}{16}}$ //

9

$\beta = \frac{\pi}{2} - 3\alpha > 0$

$\Downarrow$

$0 < \alpha < \frac{\pi}{6}$

$\tan(\alpha+\beta)$

$= \tan\left(\frac{\pi}{2} - 2\alpha\right)$

$= \frac{1}{\tan 2\alpha} > \frac{1}{\sqrt{3}} = \underline{\frac{\sqrt{3}}{3}}$ //

$5\tan(\alpha+\beta) + 4\tan(\alpha-\beta)$

$= 5\tan\left(\frac{\pi}{2} - 2\alpha\right) + 4\tan\left(4\alpha - \frac{\pi}{2}\right)$

$= \frac{5}{\tan 2\alpha} - \frac{4}{\tan 4\alpha}$

$= \frac{5}{\tan 2\alpha} - 4\cdot\frac{1-\tan^2 2\alpha}{2\tan 2\alpha}$

$= \frac{2\tan^2 2\alpha + 3}{\tan 2\alpha}$

$= 2\tan 2\alpha + \frac{3}{\tan 2\alpha}$

$\geqq 2\sqrt{2\tan 2\alpha\cdot\frac{3}{\tan 2\alpha}} = \underline{2\sqrt{6}}$ //

等号成立は $\tan 2\alpha = \frac{\sqrt{6}}{2}$ のとき

10

$y = \frac{e^x - e^{-x}}{2}$

$\updownarrow$ 逆関数 → ☆

$x = \frac{e^y - e^{-y}}{2} = \sinh y$

$\frac{dx}{dy} = \frac{e^y + e^{-y}}{2}$

$\therefore f(x) = \frac{dy}{dx} = \frac{2}{e^y + e^{-y}} = \frac{1}{\cosh y}$

ここで $\sinh y = 2$ のとき

$\cosh^2 y = 1 + \sinh^2 y = 5$

$\therefore \cosh y = \sqrt{5}$ ($\because \cosh y > 0$)

$\therefore f(2) = \frac{1}{\sqrt{5}} = \underline{\frac{\sqrt{5}}{5}}$ //

☆ $2xe^y = e^{2y} - 1$

$0 = e^{2y} - 2xe^y - 1$

$e^y = x + \sqrt{x^2+1}$

$\therefore f(x) = \frac{2}{x+\sqrt{x^2+1} + \frac{1}{x+\sqrt{x^2+1}}} = \frac{1}{\sqrt{x^2+1}}$

$f'(x) = -x(x^2+1)^{-\frac{3}{2}}$ ← 奇関数

(i) $y \neq z$ のとき

$\frac{f(y) - f(z)}{y - z} = f'(c)$

をみたす c が y と z の間に存在.

$f''(x) = -(x^2+1)^{-\frac{3}{2}} + 3x^2(x^2+1)^{-\frac{5}{2}}$

$= \cdots$

$= \frac{2x^2 - 1}{(x^2+1)^{\frac{5}{2}}}$

x	$0 \cdots \frac{1}{\sqrt{2}} \cdots$
$f''(x)$	$-\ 0\ +$
$f'(x)$	$0 \searrow -\frac{2\sqrt{3}}{9} \nearrow$

$\lim_{x\to\infty} f'(x) = 0$

より $-\frac{2\sqrt{3}}{9} \leqq f'(c) \leqq \frac{2\sqrt{3}}{9}$

$|f(y) - f(z)| \leqq a|y - z|$

をみたす a で最小は $a = \frac{2\sqrt{3}}{9}$

(ii) $y = z$ のときすべての a で成立

(i)(ii)より求める a は $\underline{\frac{2\sqrt{3}}{9}}$ //