2022 埼玉医科大(前期)

$\boxed{1}$

問1.

$$ax^3+(-4a+1)x^2+(a+1)x+6a$$
$$=(x+1)\{ax^2+(-5a+1)x+6a\}$$

(i) $x=-1$ 以外が解$\alpha, -\alpha$のとき

$$\alpha+(-\alpha)=\frac{5a-1}{a}=0$$
$$\therefore a=\frac{1}{5}$$

このときαは実数でないので不適

(ii) $x=1$を解にもつとき

$$a+(-5a+1)+6a=0$$
$$\therefore a=-\frac{1}{2}$$

$$=(x+1)\left(-\frac{1}{2}x^2+\frac{7}{2}x-3\right)$$
$$=-\frac{1}{2}(x+1)(x^2-7x+6)$$
$$=-\frac{1}{2}(x+1)(x-1)(x-6)$$

$\underline{a=\frac{-1}{2}}$ 解は $\underline{\pm1}$, $\underline{6}$

問2.

$$f(x)=3x^2+2x-a$$
$$g(x)=x^2-6x+b$$
$$\downarrow$$
$$a=\int_0^3(t^2-6t+b)\,dt$$
$$=\left[\frac{1}{3}t^3-3t^2+bt\right]_0^3$$
$$=9-27+3b$$
$$=3b-18$$

$$b=\int_1^2(3t^2+2t-a)\,dt$$
$$=\left[t^3+t^2-at\right]_1^2$$
$$=7+3-a$$
$$=10-(3b-18)$$
$$=28-3b$$

$\therefore b=\underline{7}$　$a=\underline{3}$

$\boxed{2}$

$$f'(x)=\frac{\log x}{x}$$
$$f(x)=\frac{1}{2}(\log x)^2+C$$
$$\downarrow\ f(1)=\frac{1}{2}$$
$$f(x)=\frac{1}{2}(\log x)^2+\frac{1}{2}$$

問1

$$\frac{1}{2}(\log x)^2+\frac{1}{2}=1$$
$$\iff (\log x)^2=1$$
$$\iff \log x=\pm1$$

$\therefore \alpha=\underline{e^{-1}}$, $\beta=\underline{e^{1}}$

問2

$$\int_{e^{-1}}^{e}\left\{\frac{1}{2}(\log x)^2+\frac{1}{2}\right\}dx$$
$$=\left[\frac{x}{2}(\log x)^2-x\log x+x+\frac{1}{2}x\right]_{e^{-1}}^{e}$$

$\boxed{\frac{1}{2}(\log x)^2+\log x}$ $\boxed{-\log x-1}$ $\boxed{+1}$

$$=\frac{e}{2}-e+\frac{3}{2}e-\left(\frac{e^{-1}}{2}+e^{-1}+\frac{3}{2}e^{-1}\right)$$
$$=\underline{1\cdot e+\frac{-3}{e}}$$

問3.

接点$(t, f(t))$とすると

接線 $y=\frac{\log t}{t}(x-t)+f(t)$

$\downarrow$ $(0,2)$通る

$$2=-\log t+\frac{1}{2}(\log t)^2+\frac{1}{2}$$
$$\iff 0=(\log t)^2-2\log t-3$$
$$\iff \log t=-1, 3$$
$$\therefore t=e^{-1}, e^{3}$$

$t=e^{-1}$のとき

$$y=-e\left(x-\frac{1}{e}\right)+1$$
$$=\underline{-1\cdot ex+2}$$

$t=e^{3}$のとき

$$y=\frac{3}{e^3}(x-e^3)+5$$
$$=\underline{\frac{3}{e^3}x+2}$$

$\boxed{3}$

問1.

$$d=-\tan\theta+(\sin\theta)l$$

$\underline{⑥}$　$\underline{①}$

問2.

$$d$$
$$=-\tan\theta+8\sin\theta\quad\left(0\leqq\theta<\frac{\pi}{2}\right)$$
$$d'=-\frac{1}{\cos^2\theta}+8\cos\theta$$
$$=\frac{8\cos^3\theta-1}{\cos^2\theta}$$

θ	$0\ \cdots\ \frac{\pi}{3}\ \cdots\ \frac{\pi}{2}$
d'	$+\ \ 0\ \ -\ \ \times$
d	$0\ \nearrow\ 3\sqrt{3}\ \searrow\ \times$

$\therefore \max d = \underline{3\sqrt{3}}$ //

問3.

$d = -\tan\theta + (\sin\theta)l$

$d' = \dfrac{l\cos^3\theta - 1}{\cos^2\theta}$

$d' = 0 \Leftrightarrow \cos\alpha = l^{-\frac{1}{3}}$ とする.

θ	$0 \cdots \alpha \cdots \frac{\pi}{2}$
d'	$+ \quad 0 \quad -$
d	$0 \nearrow \quad \searrow$

$\max d = -\tan\alpha + (\sin\alpha)l$

$= -\dfrac{\sqrt{1-l^{-\frac{2}{3}}}}{l^{-\frac{1}{3}}} + \sqrt{1-l^{-\frac{2}{3}}}\,l$

$= \sqrt{1-l^{-\frac{2}{3}}}\,(l - l^{\frac{1}{3}})$

$= l\sqrt{(1-l^{-\frac{2}{3}})^3}$

$= l \times (1-l^{-\frac{2}{3}})^{\frac{3}{2}}$

$= \{l^{\frac{2}{3}} \times (1-l^{-\frac{2}{3}})\}^{\frac{3}{2}}$

$= (l^{\frac{2}{3}}-1)^{\frac{3}{2}} \leqq 2$

$l^{\frac{2}{3}} - 1 \leqq 2^{\frac{2}{3}}$

$l^{\frac{2}{3}} \leqq 2^{\frac{2}{3}} + 1$

$l \leqq (2^{\frac{2}{3}}+1)^{\frac{3}{2}}$

$\therefore \max l = \underline{(1+\sqrt[3]{4})^{\frac{3}{2}}}$ //

$\boxed{4}$

問1

n(二等辺三角形)

$= 8 \times 3 = \underline{24}$ //

n(辺の長さすべて異なる)

$= {}_8C_3 - n$(二等辺三角形)

$= \underline{32}$ //

問2

P(△AHBと合同)

$= \dfrac{8}{{}_8C_3} = \underline{\dfrac{1}{7}}$ //

P(△AHCと合同)

$= \dfrac{8\times 2}{{}_8C_3} = \underline{\dfrac{2}{7}}$ //

問3

n(正八角形と2辺共有)

$+ n$(〃 1辺 〃)

$+ n$(〃 辺を共有しない)

$= 1+2+2 = \underline{5}$ //

問4. 以下の三角形と合同な確率

(1)

	△ABH	△AHC	△AHD	△AGC	△AFD
P	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

P(1回目と2回目合同)

$= \left(\frac{1}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^2$

$= \underline{\dfrac{11}{49}}$ //

(2)

$P_{\text{1回目が二等辺三角形}}$(1,2回目合同)

$= \dfrac{\left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^2}{\frac{24}{56}}$

$= \underline{\dfrac{1}{7}}$ //