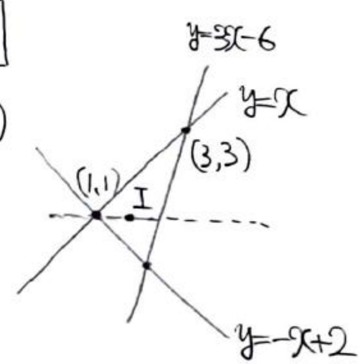
2021 東海大(医) 2日目 2/3実施

$\boxed{1}$

(1)



内心 I $(a,1)$ とおく.

$x-y=0$ と $3x-y-6=0$ までの距離が等しいので

$$\frac{|a-1|}{\sqrt{2}}=\frac{|3a-7|}{\sqrt{10}}$$

$\Leftrightarrow \sqrt{5}|a-1|=|3a-7|$

2乗すると

$5(a^2-2a+1)=9a^2-42a+49$

$\Leftrightarrow 0=4a^2-32a+44$

$\Leftrightarrow 0=a^2-8a+11$

$\therefore a=4-\sqrt{5}\quad(1<a<3)$

$\therefore \underline{(4-\sqrt{5},1)}$

(2)

$${}_{20}C_7=\frac{20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14}{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

$=20\cdot19\cdot6\cdot17\cdot2$

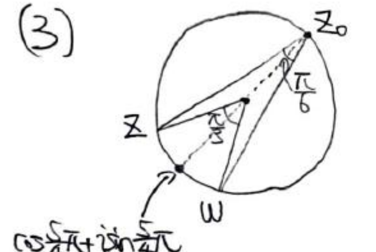
$=2^4\cdot3\cdot5\cdot17\cdot19$

$$\frac{20!}{2^{10}\cdot3^6\cdot5^4\cdot7^2}$$

$=2^{5+2+1}\cdot3^2\cdot11\cdot13\cdot17\cdot19$

最大公約数は $\underline{2^4\cdot3\cdot17\cdot19}$

(3)



図より

$z_0=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$

$=\underline{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i}$

(4) $\tan x=X$, $\tan h=H$ とする.

$f(x)$

$$=\lim_{h\to0}\frac{\frac{X+H}{1-XH}+\frac{X-H}{1+XH}-2X}{h^2}$$

$$=\lim_{h\to0}\frac{(X+H)(1+XH)+(X-H)(1-XH)-2X(1-X^2H^2)}{h^2(1-X^2H^2)}$$

$$=\lim_{h\to0}\frac{2XH^2+2X^3H^2}{h^2(1-X^2H^2)}$$

$$=\lim_{h\to0}\left(\frac{\tan h}{h}\right)^2\frac{2\tan x+2\tan^3x}{1-\tan^2x\tan^2h}$$

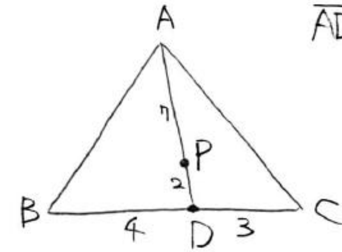
$=2\tan x+2\tan^3x$

$f\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}+\frac{2}{3\sqrt{3}}=\frac{8}{3\sqrt{3}}=\underline{\frac{8\sqrt{3}}{9}}$

(5)

$2\overrightarrow{AP}+3\overrightarrow{AP}+4\overrightarrow{AP}=3\overrightarrow{AB}+4\overrightarrow{AC}$

$\therefore \overrightarrow{AP}=\frac{7}{9}\cdot\underbrace{\frac{3\overrightarrow{AB}+4\overrightarrow{AC}}{7}}_{\overrightarrow{AD}}$



$\underline{\frac{BD}{CD}=\frac{4}{3}}$, $\underline{\frac{AP}{PD}=\frac{7}{2}}$

(6) $x^2+1=t$ とおく

$2x\,dx=dt$

x	$\sqrt{\sqrt{e}-1}\to\sqrt{e^2-1}$
t	$\sqrt{e}\to e^2$

(与式)

$$=\int_{\sqrt{e}}^{e^2}\frac{1}{2}\cdot\frac{\log(\log t)}{t}dt$$

$\log t=u$

$$=\int_{\frac{1}{2}}^{2}\frac{1}{2}\log u\,du$$

$$=\frac{1}{2}\Big[u\log u-u\Big]_{\frac{1}{2}}^{2}$$

$$=\frac{1}{2}\left\{2\log2-2-\frac{1}{2}\log\frac{1}{2}+\frac{1}{2}\right\}$$

$$=\frac{1}{2}\left(\frac{5}{2}\log2-\frac{3}{2}\right)$$

$=\underline{\frac{5}{4}\log2-\frac{3}{4}}$

$\boxed{2}$

(1)

(i) $\underline{x=\frac{1}{3}}$ で最大値 $\underline{\frac{3}{2}}$

(ii)

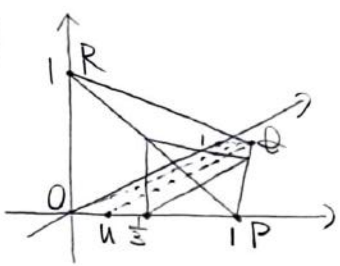
y'

$$=\frac{5x^2-(5x-1)2x}{x^4}$$

$$=\frac{-5x+2}{x^3}$$

$\underline{x=\frac{2}{5}}$ で最大値 $\underline{\frac{25}{4}}$

(2)



(i) RP: $z=-x+1$

QP: $y=\frac{-1}{1-u}(x-1)$

上にそれぞれ $x=\frac{1}{3}$ を代入すると

$z=\frac{2}{3}$, $y=\frac{2}{3(1-u)}$

$\therefore S=\frac{2}{3}\cdot\frac{2}{3(1-u)}\cdot\frac{1}{2}$

$=\frac{2}{9(1-u)}$

(1)(i)より $u=\underline{\frac{1}{3}}$ で最大値 $\underline{\frac{1}{3}}$

V

$=\frac{2}{9(1-u)}\times\frac{2}{3}\times\frac{1}{3}$

$=\frac{4}{81(1-u)}$

W

$=1\cdot1\cdot\frac{1}{2}\cdot1\cdot\frac{1}{3}-V$

$=\frac{1}{6}-V$

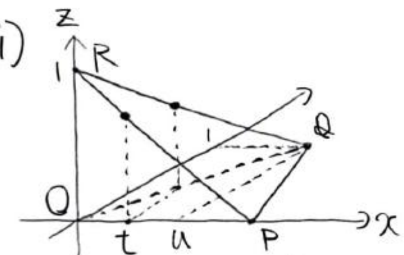
$3V=2W=2\left(\frac{1}{6}-V\right)$

$\Leftrightarrow 5V=\frac{1}{3}$

$\Leftrightarrow V=\frac{4}{81(1-u)}=\frac{1}{15}$

$\Leftrightarrow \frac{20}{27}=1-u$ $\quad\therefore \underline{u=\frac{7}{27}}$

(ii)



直線QR: $\begin{cases} x=-uk \\ y=-k \quad (k\in\mathbb{R}) \\ z=1+k \end{cases}$

とおる $x=t$ のとき $k=\frac{t}{-u}$.

HとQRの交点は

$\underline{\left(t,\frac{t}{u},1-\frac{t}{u}\right)}$

(iii)

S

$=\left\{(1-t)+\left(1-\frac{t}{u}\right)\right\}\frac{t}{u}\cdot\frac{1}{2}$

$=\left(2-t-\frac{t}{u}\right)\frac{t}{2u}$ $\quad t=\frac{1}{3}$

$=\left(\frac{5}{3}-\frac{1}{3u}\right)\frac{1}{6u}$

$=\frac{1}{18}\cdot\frac{5u-1}{u^2}$

(1)(ii)より $u=\underline{\frac{2}{5}}$ で最大値 $\underline{\frac{25}{72}}$

をとる.

W

$=\int_0^{\frac{1}{3}}\left(2-t-\frac{t}{u}\right)\frac{t}{2u}dt$

$=\int_0^{\frac{1}{3}}\left\{\frac{t}{u}-\frac{1}{2u}\left(1+\frac{1}{u}\right)t^2\right\}dt$

$=\left[\frac{t^2}{2u}-\frac{1}{6u}\left(1+\frac{1}{u}\right)t^3\right]_0^{\frac{1}{3}}$

$=\frac{8u-1}{162u^2}$

$V=W$ のとき $W=\frac{1}{12}$ より

$\left(\because V+W=\frac{1}{6}\right)$

$\frac{8u-1}{162u^2}=\frac{1}{12}$

$\Leftrightarrow 27u^2-16u+2=0$

$\therefore \underline{u=\frac{8+\sqrt{10}}{27}}$

$\boxed{3}$

(1)

$f_{n+1}(x)$

$=\dfrac{\frac{a_{n-1}}{x+2}+a_n}{\frac{b_{n-1}}{x+2}+b_n}=\dfrac{a_nx+2a_n+a_{n-1}}{b_nx+2b_n+b_{n-1}}$

$\therefore a_{n+1}=2a_n+a_{n-1}$

$b_{n+1}=2b_n+b_{n-1}$

$f_2(x)=\dfrac{\frac{1}{x+2}+3}{\frac{1}{x+2}+2}$

$=\frac{3x+7}{2x+5}$

$\therefore \underline{a_2=7,\ b_2=5}$

$a_3=2a_2+a_1=\underline{17}$

$b_3=2b_2+b_1=\underline{12}$

(2)

(a) $a_{n+2}=\underline{2}a_{n+1}+\underline{1}\cdot a_n$

(b) $b_{n+2}=\underline{2}b_{n+1}+\underline{1}\cdot b_n$

(3)

特 $\alpha^2-2\alpha-1=0$

$\therefore \alpha=\underline{1+\sqrt{2}}$, $\beta=\underline{1-\sqrt{2}}$

$a_n=(a_2-\alpha a_1)\beta^{n-1}$

$=\{7-(1+\sqrt{2})3\}\beta^{n-1}$

$=\underline{(4-3\sqrt{2})\beta^{n-1}}$

$a_n = シ(1+\sqrt{2})^{n-1} + ス\cdot(1-\sqrt{2})^{n-1}$ とおく

$\downarrow$ $n=1,2$を代入

$a_1 = シ+ス = 3$

$a_2 = シ(1+\sqrt{2}) + ス(1-\sqrt{2}) = 7$

$-)\ シ(1+\sqrt{2}) + ス(1+\sqrt{2}) = 3+3\sqrt{2}$

$-2\sqrt{2}\cdot ス = 4-3\sqrt{2}$

$\therefore ス = \frac{3}{2} - \sqrt{2}$

$シ = \frac{3}{2} + \sqrt{2}$ //

$b_n = セ(1+\sqrt{2})^{n-1} + ソ(1-\sqrt{2})^{n-1}$ とおく

$\downarrow$ $n=1,2$を代入

$b_1 = セ+ソ = 2$

$b_2 = セ(1+\sqrt{2}) + ソ(1-\sqrt{2}) = 5$

$-)\ セ(1+\sqrt{2}) + ソ(1+\sqrt{2}) = 2+2\sqrt{2}$

$-2\sqrt{2}\cdot ソ = 3-2\sqrt{2}$

$\therefore ソ = 1 - \frac{3\sqrt{2}}{4}$

$セ = 1 + \frac{3\sqrt{2}}{4}$ //

(4)

$f_n(0) = \frac{a_n}{b_n}$

$$\lim_{n\to\infty} f_n(0) = \lim_{n\to\infty} \frac{シ + ス\left(\frac{\beta}{\alpha}\right)^{n-1}}{セ + ソ\left(\frac{\beta}{\alpha}\right)^{n-1}} = \frac{\frac{3}{2}+\sqrt{2}}{1+\frac{3}{4}\sqrt{2}} = \sqrt{2}$$ //