# 2020 東海大(医) 2日目 (2/3)
赤本掲載

## $\boxed{1}$

(1)

$0.101_{(2)}$

$= 1\times\frac{1}{2}+0\times\frac{1}{2^2}+1\times\frac{1}{2^3}$

$= \frac{5}{8} = \underline{0.625}$

(2) $f(x)=x\log x$

$g(x)=xf(x)$

(与式)

$= \lim_{h\to 0}\frac{g(e^2+h)-g(e^2)}{h}$

$= g'(e^2)$

$= f(e^2)+e^2f'(e^2)$

$= 2e^2+e^2(2+1)$

$= \underline{5e^2}$ …イ

(与式)

$= \lim_{n\to\infty}\frac{e-1}{n}\sum_{k=1}^{n}\left(1+\frac{k(e-1)}{n}\right)f\left(1+\frac{k(e-1)}{n}\right)$

$= (e-1)\int_0^1\{1+(e-1)x\}f(1+(e-1)x)dx$

$\downarrow$ $1+(e-1)x=t$, $(e-1)dx=dt$

$= \int_1^e tf(t)dt$

$= \int_1^e t^2\log t\,dt$

$= \left[\frac{t^3}{3}\log t-\frac{t^3}{9}\right]_1^e$

$\boxed{t^2\log t+\frac{t^2}{3}}$ $\boxed{-\frac{t^2}{3}}$

$= \frac{e^3}{3}-\frac{e^3}{9}+\frac{1}{9}$

$= \underline{\frac{2}{9}e^3+1}$ …ウ

(3)

$f(x)$を$x-1$で割った余りは

$f(1)$ ← 剰余の定理

$= 4+5a+a^2+3-a$

$= a^2+4a+7$

$= (a+2)^2+3=R$

$\therefore \underline{R\geqq 3}$

(4)

オ+カ$\sqrt{キ}$

$= (\sqrt{3}-\sqrt{2})^{-2}$

$= \frac{1}{5-2\sqrt{6}}$

$= \underline{5+2\sqrt{6}}$

(5)

$\tan(x+y)+\tan(x-y)$

$= \frac{\tan x+\tan y}{1-\tan x\tan y}+\frac{\tan x-\tan y}{1+\tan x\tan y}$

$= \frac{3}{2}(\tan x+\tan y)+\frac{3}{4}(\tan x-\tan y)$

$= \frac{9}{4}\tan x+\frac{3}{4}\tan y$

$= \frac{9}{4}\tan x+\frac{3}{4}\cdot\frac{1}{3\tan x}$

$= \frac{1}{4}\left(9\tan x+\frac{1}{\tan x}\right)$

$\geqq \frac{1}{4}2\sqrt{9\tan x\cdot\frac{1}{\tan x}}$ $\boxed{(相加平均)\geqq(相乗平均)}$

$= \frac{3}{2}$

等号成立は $\tan x=\frac{1}{3}$ のとき

このとき最小値 $\underline{\frac{3}{2}}$

## $\boxed{2}$

(1)

$g(1964)$

$= |1964-4691|$

$= \underline{2727}$

(2)

$g(x)=0 \iff f(x)$が左右対称

$1\sim 10000$で左右対称なのは

1ケタ … 9コ

2ケタ … 9コ ← 11~99のゾロ目

1?1 … 10コ ⟲

3ケタ … 10×9=90コ

4ケタ … 90コ ← 10~99

合計は 9+9+90+90= $\underline{198コ}$

(3)

$g(x)$の00次の値は $\underline{9}$

$g(x)=99$となる最小の自然数$x$は $\underline{100}$

$g(x)=99$になるのは

1?0 … 10コ

0?1 … 10コ ← 1ケタになってしまう

⋮

8?9 … 10コ

9?8 … 10コ

合計は 20×8−10= $\underline{170コ}$

$g(x)=99$の次は

$x=1131$などで $g(x)=\underline{180}$

(4)

$x=abcdef$とする.

$g(x)$

$=|abcdef-fedcba|$

$=|99999(a-f)+9990(b-e)+900(c-d)|$

$=9|11111(a-f)+1110(b-e)+100(c-d)|$

$=196398$

↓

$|11111(a-f)+1110(b-e)+100(c-d)|=21822$

(i) $a-f=2$のとき

$1110(b-e)+100(c-d)=-400$

$\therefore b=e,\ c-d=-4$

(ii) $a-f=-2$のとき

$1110(b-e)+100(c-d)=400$

$\therefore b=e,\ c-d=4$

最小は(ii), 最大は(i)のとき

$\underline{最小値 104003, 最大値 995997}$

$\boxed{3}$

(1)

$P_k=\left(\frac{2}{5}\right)^{k-1}\frac{2}{5}=\underline{\left(\frac{2}{5}\right)^k}$

$\sum_{k=1}^{\infty}P_k=\frac{\frac{2}{5}}{1-\frac{2}{5}}=\underline{\frac{2}{3}}$

(2)

$Q_k$

$=P$(Bが1回も勝たない)

$=P(\underbrace{\qquad}_{A\times 1}A)$

$=(k-1)\left(\frac{2}{5}\right)^k$

$\sum_{k=1}^{\infty}Q_k=S$とおく

$$
\begin{aligned}
S&=1\cdot\left(\tfrac{2}{5}\right)^2+2\cdot\left(\tfrac{2}{5}\right)^3+\cdots\\
-)\ \tfrac{2}{5}S&=\qquad\quad 1\cdot\left(\tfrac{2}{5}\right)^3+\cdots\\
\hline
\tfrac{3}{5}S&=\left(\tfrac{2}{5}\right)^2+\left(\tfrac{2}{5}\right)^3+\cdots
\end{aligned}
$$

$=\frac{\frac{4}{25}}{1-\frac{2}{5}}=\frac{4}{15}$

$\therefore S=\frac{4}{15}\cdot\frac{5}{3}=\underline{\frac{4}{9}}$

(3)

$R_k$

$=P$(Bが1回も勝たない)

$+P$(Bが1回勝う)

$=P$(引1〜引1A)

$+P(\underbrace{\qquad}_{B\times 1}A)$

$=\left(\frac{2}{5}\right)^{k-1}\frac{1}{5}+(k-1)\left(\frac{2}{5}\right)^{k-1}\frac{1}{5}$

$=\frac{k}{5}\left(\frac{2}{5}\right)^{k-1}$

$\sum_{k=1}^{\infty}R_k=W$とする.

$$
\begin{aligned}
W&=\tfrac{1}{5}+\tfrac{2}{5}\left(\tfrac{2}{5}\right)+\tfrac{3}{5}\left(\tfrac{2}{5}\right)^2+\cdots\\
-)\ \tfrac{2}{5}W&=\qquad\ \tfrac{1}{5}\left(\tfrac{2}{5}\right)+\tfrac{2}{5}\left(\tfrac{2}{5}\right)^2+\cdots\\
\hline
\tfrac{3}{5}W&=\tfrac{1}{5}+\tfrac{1}{5}\cdot\tfrac{2}{5}+\tfrac{1}{5}\left(\tfrac{2}{5}\right)^2+\cdots
\end{aligned}
$$

$=\frac{\frac{1}{5}}{1-\frac{2}{5}}=\frac{1}{3}$

$\therefore W=\underline{\frac{5}{9}}$

(4)

$$
\begin{aligned}
a_3&=\sum_{k=3}^{\infty}(k-1)(k-2)p^k\\
-)\ pa_3&=\sum_{k=3}^{\infty}(k-1)(k-2)p^{k+1}\\
\hline
(1-p)a_3&=2p^3+4p^4+6p^5+8p^6+\cdots\\
-)\ (1-p)pa_3&=\qquad\ 2p^4+4p^5+\cdots\\
\hline
(p^2-2p+1)a_3&=2p^3+2p^4+2p^5+\cdots
\end{aligned}
$$

$\iff(p-1)^2a_3=\frac{2p^3}{1-p}$

$\therefore a_3=\underline{\frac{2p^3}{(1-p)^3}}$

$$
\begin{aligned}
a_n&=\sum_{k=n}^{\infty}(k-1)(k-2)\cdots\{k-(n-1)\}p^k\\
-)\ pa_n&=\sum_{k=n}^{\infty}(k-1)(k-2)\cdots\{k-(n-1)\}p^{k+1}\\
\hline
(1-p)a_n&=(n-1)!p^n+(n-1)(n-1)!p^{n+1}+(n-1)n!p^{n+2}+\cdots
\end{aligned}
$$

$=(n-1)p\{(n-2)!p^{n-1}+(n-1)!p^n+n!p^{n+1}+\cdots\}$

$=(n-1)pa_{n-1}$

$\iff\frac{a_n}{(n-1)!}=\frac{p}{1-p}\frac{a_{n-1}}{(n-2)!}$

$=\cdots$

$=\left(\frac{p}{1-p}\right)^{n-1}\frac{a_1}{0!}$

$=\left(\frac{p}{1-p}\right)^n$

$\therefore \underline{a_n=(n-1)!\left(\frac{p}{1-p}\right)^n}$

(5)

$S_k$

A×2, B×1

$$= P(\sim\sim A)$$

$$= \frac{(k-1)!}{2!\,1!\,(k-4)!}\left(\frac{2}{5}\right)^{k-1}\frac{1}{5}$$

$$= \frac{(k-1)(k-2)(k-3)}{4}\left(\frac{2}{5}\right)^{k}$$

$$= \frac{1}{4}a_4\Big|_{p=\frac{2}{5}}$$

$$= \frac{1}{4}\cdot 3!\left(\frac{\frac{2}{5}}{\frac{3}{5}}\right)^4$$

$$= \frac{3}{2}\left(\frac{2}{3}\right)^4$$

$$= \left(\frac{2}{3}\right)^3$$

$$= \frac{8}{27}$$