2020 獨協医科（医）

1

(1)

$7x^2+4\sqrt{3}xy+3y^2$

$=r^2(7\cos^2\theta+4\sqrt{3}\sin\theta\cos\theta+3\sin^2\theta)$

$=r^2(4\cos 2\theta+2\sqrt{3}\sin 2\theta+3)$

$=r^2(2\cos 2\theta+2\sqrt{3}\sin 2\theta+5)$

$=r^2\{4\sin(2\theta+\frac{\pi}{6})+5\}=4$

$(\frac{\pi}{6}\leqq 2\theta+\frac{\pi}{6}<\frac{25}{6}\pi)$

$2\theta+\frac{\pi}{6}=\frac{3}{2}\pi, \frac{7}{2}\pi$

$\Leftrightarrow \theta=\frac{2}{3}\pi, \frac{5}{3}\pi$ のとき

最大値 $r=2$　$\leftarrow$ $r^2=4$

$2\theta+\frac{\pi}{6}=\frac{\pi}{2}, \frac{5}{2}\pi$

$\Leftrightarrow \theta=\frac{1}{6}\pi, \frac{7}{6}\pi$ のとき

最小値 $r=\frac{2}{3}$　$\leftarrow$ $9r^2=4$

(2)

H と l を連立

$x^2-\{k(x-1)+3\}^2=1$

$\Leftrightarrow x^2-k^2(x-1)^2-6k(x-1)-10=0$

$\Leftrightarrow (1-k^2)x^2+(2k^2-6k)x-k^2+6k-10=0$ …☆

$\frac{D}{4}$　$(k\neq\pm1)$

$=(k^2-3k)^2-(1-k^2)(-k^2+6k-10)$

$=k^4-6k^3+9k^2+(1-k^2)(k^2-6k+10)$

$=-6k^3+9k^2+k^2-6k+10+6k^3-10k^2$

$=-6k+10=0$

$\therefore k=\frac{5}{3}$

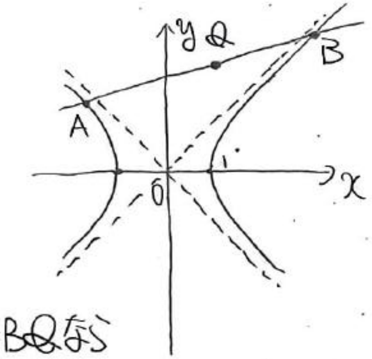
$k=\pm1$ のときは☆が1次方程式になるのでただ1つ共有点をもつ.

$\therefore k=\pm1, \frac{5}{3}$

☆で異なる2つの実数解をもつとき

$\alpha+\beta=-\frac{2k^2-6k}{1-k^2}$

$=\frac{2k^2-6k}{k^2-1}$



AQ=BQ なら

図より A と B の中点が Q.

$\frac{\alpha+\beta}{2}=1$

$\Leftrightarrow \frac{2k^2-6k}{k^2-1}=2$　$\therefore k=\frac{1}{3}$

2

$2^{x+y}=u$ とおく. $t=u+u^{-1}$.

$t^3=u^3+3u+3u^{-1}+u^{-3}$

$\Leftrightarrow u^3+u^{-3}=t^3-3t$

$t^2=u^2+2+u^{-2}$

$\Leftrightarrow u^2+u^{-2}=t^2-2$

(与式)

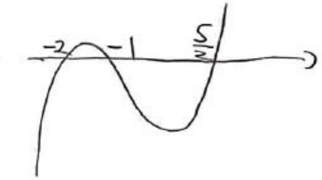
$=2(u^3+u^{-3})+u^2+u^{-2}-5t-8$

$=2(t^3-3t)+t^2-2-5t-8$

$=2t^3+t^2-11t-10\leqq 0$

$\Leftrightarrow (t+1)(2t^2-t-10)\leqq 0$

$\Leftrightarrow (t+1)(2t-5)(t+2)\leqq 0$



(相加平均)≧(相乗平均)より

$t\geqq 2\sqrt{u\cdot u^{-1}}=2$

図も考慮して

$2\leqq t\leqq\frac{5}{2}$

$x+y=\pm1$ のとき $t=\frac{5}{2}$.

$\therefore -1\leqq x+y\leqq 1$

② で $S=\log_2 x$ とおくと

$S^2\leqq 2S-\frac{\log_2 8}{S}$

$\Leftrightarrow S^4-2S^3+3S\leqq 0$　$(S\neq 0)$

$\Leftrightarrow S(S^3-2S^2+3)\leqq 0$　$(S\neq 0)$

$\Leftrightarrow S(S+1)(S^2-3S+3)\leqq 0$　$(S\neq 0)$　（$S^2-3S+3\geqq 0$）

$\Leftrightarrow S(S+1)\leqq 0$　$(S\neq 0)$

$\therefore -1\leqq S<0$

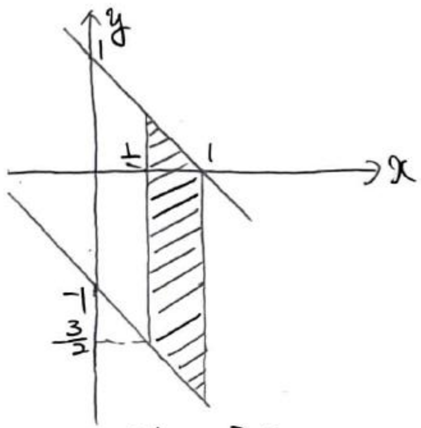
$\therefore \frac{1}{2}\leqq x<1$

$3x+2y=k$ とおく.

$\Leftrightarrow y=-\frac{3}{2}x+\frac{k}{3}$

①: $-x-1 \leqq y \leqq -x+1$

②: $\frac{1}{2} \leqq x < 1$



$(x, y)=\left(\frac{1}{2}, -\frac{3}{2}\right)$ のとき

$\min k=\frac{3}{2}-3=\frac{-3}{2}$ //

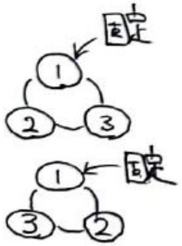
3

(1)

(i) P(同じ番号が隣)

$=\frac{2!\times 2^3}{5!}$

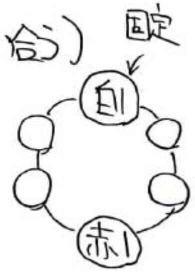
$=\frac{16}{120}=\frac{2}{15}$ //



(ii) P(白1と赤1が向かい合う)

$=\frac{4!}{5!}$

$=\frac{1}{5}$ //

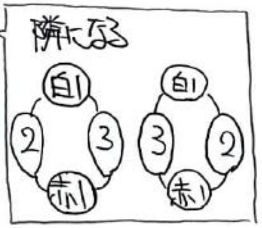


P(白1赤1向かい合う∩同じ番号隣になる)

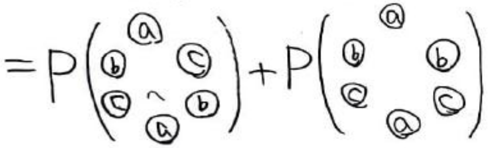
$=\frac{4!-2\cdot 2\cdot 2^2}{5!}$

$=\frac{16}{120}$

$=\frac{2}{15}$ //



P(どの番号も隣り合わない)



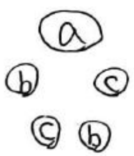
$=\frac{2\times 2^3}{5!}+\frac{2\times 2^3}{5!}$

$=\frac{4}{15}$ //

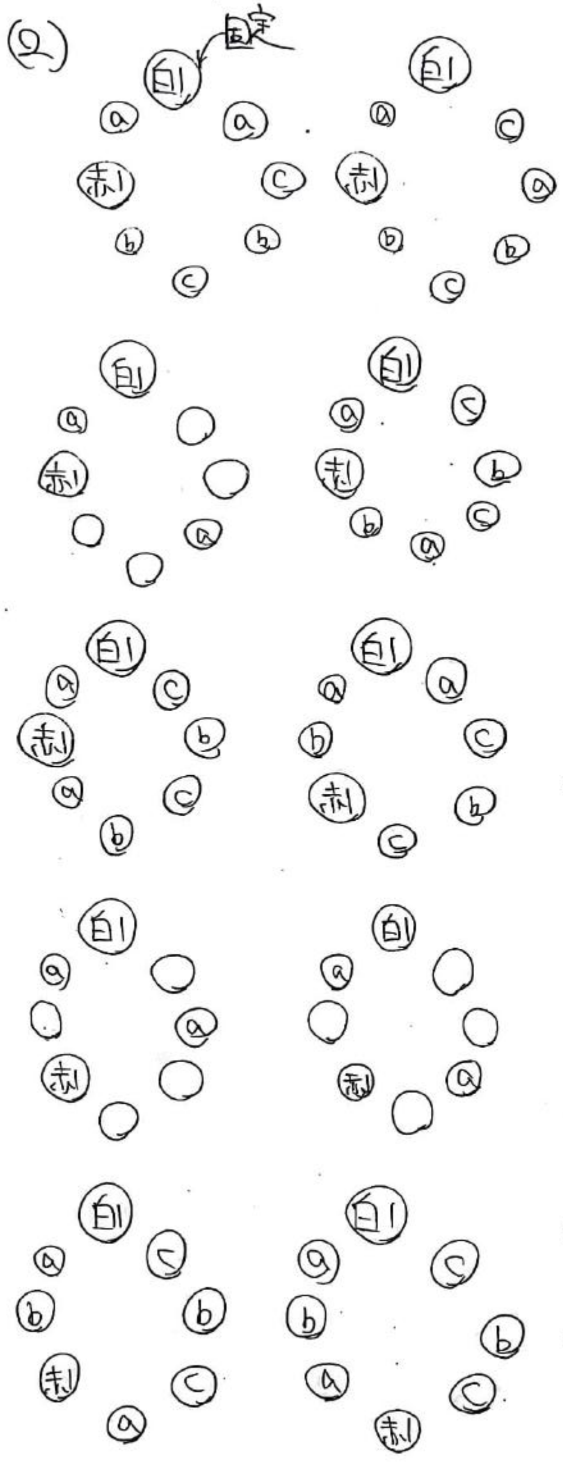
P(同じ番号が1組隣り合う)

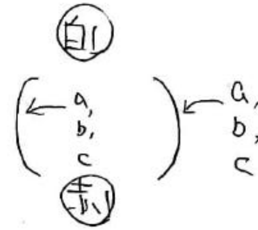
$=\frac{3\cdot 2\cdot 2\cdot 2^2}{5!}$

$=\frac{48}{120}=\frac{2}{5}$ //

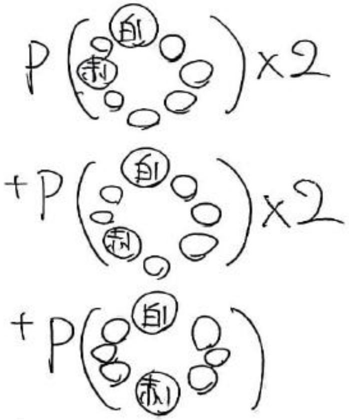


(2)





求める確率は



$=\frac{3!2^3+3!2^3+2\cdot 3!2^3+3!2^3+3!2^3}{7!}\times 2$

$+\frac{3!2^3+2\cdot 3!2^3+2\cdot 3!2^3+3!2^3}{7!}\times 2$

$+\frac{3!2^3+3!3!2^3}{7!}$

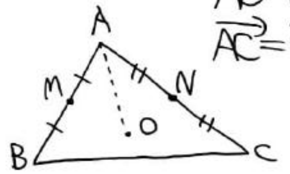
$=\frac{3!2^3}{7!}(12+12+7)$

$=\frac{8}{7\cdot 6\cdot 5\cdot 4}\cdot 31$

$=\frac{31}{105}$ //

4

(1)



$\overrightarrow{AB}=\vec{b}$
$\overrightarrow{AC}=\vec{c}$ とする.

$\overrightarrow{AO}=s\vec{b}+t\vec{c}$ とおく

$\overrightarrow{MO}\cdot\vec{b}$
$=(\overrightarrow{AO}-\overrightarrow{AM})\cdot\vec{b}$
$=\left\{\left(s-\frac{1}{2}\right)\vec{b}+t\vec{c}\right\}\cdot\vec{b}$
$=16\left(s-\frac{1}{2}\right)+5t=0$
$\Leftrightarrow 16s+5t=8 \cdots ①$

$\overrightarrow{NO}\cdot\vec{c}$
$=\left\{s\vec{b}+\left(t-\frac{1}{2}\right)\vec{c}\right\}\cdot\vec{c}$
$=5s+25\left(t-\frac{1}{2}\right)=0$
$\Leftrightarrow s+5t=\frac{5}{2} \cdots ②$

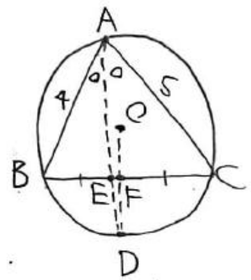
①−②より
$15s=\frac{11}{2}$ $\therefore s=\frac{11}{30}$
$5t=\frac{75}{30}-\frac{11}{30}=\frac{64}{30}=\frac{32}{15}$
$\therefore t=\frac{32}{75}$

$\therefore \overrightarrow{AO}=\frac{11}{30}\overrightarrow{AB}+\frac{32}{75}\overrightarrow{AC}$

$\overrightarrow{AO}\cdot\overrightarrow{AB}=\frac{11}{30}\cdot16+\frac{32}{75}\cdot5$
$=\frac{11}{15}\cdot8+\frac{32}{15}$
$=8$

※ 本来は
$\overrightarrow{AO}\cdot\overrightarrow{AB}=|\overrightarrow{AO}||\overrightarrow{AB}|\cos\angle OAM$
$=|\overrightarrow{AB}||\overrightarrow{AM}|$
$=4\cdot2=8$
が良いと思います.

(2)



$\overset{\frown}{BD}=\overset{\frown}{DC}$ より
$\angle BAD=\angle CAD$
$\therefore BE:EC=4:5$
$\overrightarrow{AE}=\frac{5}{9}\overrightarrow{AB}+\frac{4}{9}\overrightarrow{AC}$

$|\overrightarrow{BC}|^2=|\vec{c}-\vec{b}|^2$
$=25-2\cdot5+16=31$

$\therefore BC=\sqrt{31}$
$\therefore BE=\frac{4}{9}\sqrt{31}$, $EC=\frac{5}{9}\sqrt{31}$

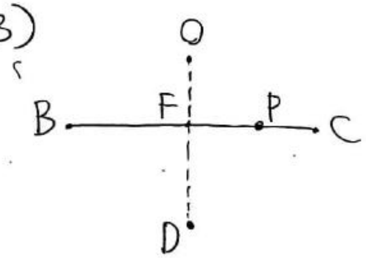
$|\overrightarrow{AE}|^2=\frac{25}{81}\cdot16+\frac{40}{81}\cdot5+\frac{16}{81}\cdot25$
$=\frac{1000}{81}$

$\therefore AE=\frac{10\sqrt{10}}{9}$

方べき
$BE\cdot EC=AE\cdot ED$
$\Leftrightarrow \frac{20}{81}\cdot31=\frac{10}{9}\sqrt{10}\cdot ED$
$\Leftrightarrow ED=\frac{2}{9}\cdot31\cdot\frac{1}{\sqrt{10}}=\frac{62\sqrt{10}}{90}$

$AE:ED=100:62=50:31$
$\overrightarrow{AD}=t\overrightarrow{AE}$ のとき $t=\frac{81}{50}$

(3)



$\cos\angle BAC=\frac{1}{4}$
$\sin\angle BAC=\frac{\sqrt{15}}{4}$

正 $\sqrt{31}\cdot\frac{4}{\sqrt{15}}=2R$

$\therefore R=\frac{2\sqrt{31}}{\sqrt{15}}$

方べき
$BF\cdot FC=FD\cdot(R+OF)$
$\Leftrightarrow \frac{BC}{2}\cdot\frac{BC}{2}=(R-OF)(R+OF)$
$\Leftrightarrow \frac{31}{4}=\frac{124}{15}-OF^2$
$\Leftrightarrow OF^2=\frac{124\cdot4}{60}-\frac{31\cdot15}{60}$
$=\frac{31}{60}$

$\therefore OF=\frac{\sqrt{31}}{2\sqrt{15}}$

$FP=\alpha$ とすると

三平方
$OD^2=OP^2+PD^2$
$\Leftrightarrow \frac{124}{15}=OF^2+\alpha^2+FD^2+\alpha^2$
$\Leftrightarrow \frac{124}{15}=\frac{31}{60}+2\alpha^2+\frac{31}{60}\cdot9$
$\Leftrightarrow \frac{248}{30}=\frac{31\cdot5}{30}+2\alpha^2$
$\Leftrightarrow 2\alpha^2=\frac{93}{30}$

$\therefore \alpha=\frac{\sqrt{93}}{\sqrt{60}}=\frac{\sqrt{3}\sqrt{31}}{2\sqrt{15}}$

$BP=BF+\alpha$
$CP=BF-\alpha$
$\frac{CP}{BP}=\frac{BF-\alpha}{BF+\alpha}=\frac{3-\sqrt{5}}{2}$

5

(1)

$0 \leqq x < 1$ のとき

$f(x) = 0$ //

$1 \leqq x < 2$ のとき

$f(x) = \left(\frac{1}{2} - 0\right)\left|\cos\frac{\pi}{2}x\right|$

$= \frac{-1}{2}\cos\frac{\pi}{2}x$ //

$2n-2 \leqq x < 2n-1$ のとき

$f(x) = (n-1-(n-1))\left|\cos\frac{\pi}{2}x\right|$

$= 0$

$2n-1 \leqq x < 2n$ のとき

$f(x) = \left(n-\frac{1}{2}-(n-1)\right)\left|\cos\frac{\pi}{2}x\right|$

$= \frac{1}{2}\left|\cos\frac{\pi}{2}x\right|$

$V_n = \int_{2n-2}^{2n} f(x)^2 \pi\, dx$

$= \int_{2n-1}^{2n} \pi \cdot \frac{1}{4}\cos^2\frac{\pi}{2}x\, dx$

$= \frac{\pi}{8}\int_{2n-1}^{2n} (1+\cos\pi x)\, dx$

$= \frac{\pi}{8}\left[x + \frac{1}{\pi}\sin\pi x\right]_{2n-1}^{2n}$

$= \frac{\pi}{8}$

$\therefore V_1 = \frac{\pi}{8}$

$\sum_{k=1}^{4n} V_k = \frac{\pi}{8}\cdot 4n = \frac{\pi n}{2}$ //

(2)

$I = \int e^{-x}\cos\frac{\pi}{2}x\, dx$

$= -e^{-x}\cos\frac{\pi}{2}x - \int(-e^{-x})\left(-\frac{\pi}{2}\right)\sin\frac{\pi}{2}x\, dx$

$= -e^{-x}\cos\frac{\pi}{2}x - \frac{\pi}{2}\int e^{-x}\sin\frac{\pi}{2}x\, dx$

$= -e^{-x}\cos\frac{\pi}{2}x$

$-\frac{\pi}{2}\left\{-e^{-x}\sin\frac{\pi}{2}x - \int(-e^{-x})\frac{\pi}{2}\cos\frac{\pi}{2}x\, dx\right\}$

$= -e^{-x}\cos\frac{\pi}{2}x + \frac{\pi}{2}e^{-x}\sin\frac{\pi}{2}x$

$-\frac{\pi^2}{4}\int e^{-x}\cos\frac{\pi}{2}x\, dx$

$\left(1+\frac{\pi^2}{4}\right)I = \frac{e^{-x}}{2}\left(\pi\sin\frac{\pi}{2}x - 2\cos\frac{\pi}{2}x\right)$

$\therefore I = \frac{2e^{-x}}{\pi^2+4}\left(\pi\sin\frac{\pi}{2}x - 2\cos\frac{\pi}{2}x\right)$ //

(3)

T_n

$= \int_{2n-1}^{2n} e^{-x}\frac{1}{2}\left|\cos\frac{\pi}{2}x\right| dx$

$= \int_{2n-1}^{2n} \frac{e^{-x}}{2}(-1)^n\cos\frac{\pi}{2}x\, dx$

$= \frac{(-1)^n}{2}\int_{2n-1}^{2n} e^{-x}\cos\frac{\pi}{2}x\, dx$

$= \frac{(-1)^n}{2}\left[\frac{2e^{-x}}{\pi^2+4}\left(\pi\sin\frac{\pi}{2}x - 2\cos\frac{\pi}{2}x\right)\right]_{2n-1}^{2n}$

$= \frac{(-1)^n}{\pi^2+4}\left\{e^{-2n}\cdot 2\cos n\pi - e^{-2n+1}\cdot\pi\sin\left(n-\frac{1}{2}\right)\pi\right\}$

$= \frac{(-1)^n}{\pi^2+4}\left\{-2e^{-2n}(-1)^n - e^{-2n+1}\cdot\pi\cdot(-1)^{n-1}\right\}$

$= \frac{e^{-2n}}{\pi^2+4}(-2+\pi e)$

$= \frac{\pi e-2}{\pi^2+4}e^{-2n}$　　$\therefore T_1 = \frac{1}{\pi^2+4}\left(\frac{\pi}{e} - \frac{2}{e^2}\right)$ //

$\sum_{n=1}^{\infty} T_n = \frac{1}{1-e^{-2}}\cdot\frac{1}{\pi^2+4}\left(\frac{\pi}{e} - \frac{2}{e^2}\right)$

$= \frac{1}{\pi^2+4}\cdot\frac{e}{e-e^{-1}}\cdot\frac{\pi e-2}{e^2}$

$= \frac{e\pi-2}{(\pi^2+4)(e^2-1)}$ //