2019 東海大(医) 2/3 2日目

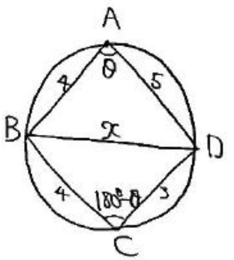
1

(1) $\lim_{n\to\infty}\left\{\left(\frac{1}{3}\right)^n+\left(\frac{2}{3}\right)^n\right\}^{\frac{1}{n}}$

$=\lim_{n\to\infty}\left\{\left\{\left(\frac{1}{2}\right)^n+1\right\}\left(\frac{2}{3}\right)^n\right\}^{\frac{1}{n}}$

$=\lim_{n\to\infty}\left\{\left(\frac{1}{2}\right)^n+1\right\}^{\frac{1}{n}}\frac{2}{3}=\underline{\frac{2}{3}}$ //

(2)



$\triangle ABD$と$\triangle BCD$に余

$$\begin{cases} x^2=16+25-40\cos\theta \\ x^2=16+9-24\underbrace{\cos(180^\circ-\theta)}_{-\cos\theta} \end{cases}$$

$\downarrow$

$41-40\cos\theta=25+24\cos\theta$

$\Leftrightarrow 16=64\cos\theta$

$\therefore \cos\theta=\frac{1}{4}$

$x^2=31 \quad \therefore x=BD=\underline{\sqrt{31}}$ //

(四角形ABCD)

$=\frac{1}{2}\cdot4\cdot5\underbrace{\sin\theta}_{\frac{\sqrt{15}}{4}}+\frac{1}{2}\cdot4\cdot3\underbrace{\sin(180^\circ-\theta)}_{\frac{\sqrt{15}}{4}}$

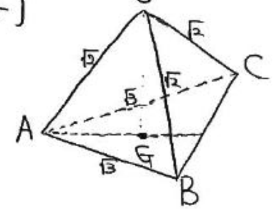
$=\frac{\sqrt{15}}{8}(20+12)=\underline{4\sqrt{15}}$ //

(3) $\sum_{n=1}^{99} n!$

$=\underbrace{1!}_{1}+\underbrace{2!}_{2}+\underbrace{3!}_{6}+\underbrace{4!}_{24}+\underbrace{5!+6!\cdots+99!}_{ここは一の位が0}$

これの一の位は$\underline{3}$ //

(4)



$\overrightarrow{OA}\cdot\overrightarrow{OG}$

$=\vec{a}\cdot\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$=\frac{1}{3}(2+\sqrt{2}\sqrt{2}\cos\angle AOB+\sqrt{2}\sqrt{2}\cos\angle AOC)$

$=\frac{1}{3}\left(2+\frac{2+2-3}{2}+\frac{2+2-3}{2}\right)$

$=\underline{1}$ //

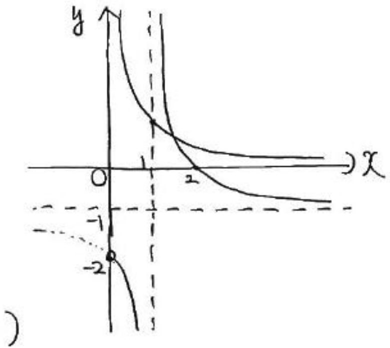
(5) b. d. e

解答欄が2つのみなので

出題ミスでしょう。

2

$C_1: y=\frac{1}{x} \quad C_2: y=\frac{1}{x-1}-1$



(1)

C_1の接点を$(s,\frac{1}{s})$

C_2の接点を$(t,\frac{1}{t-1}-1)$とおくと

接線: $y=-\frac{1}{s^2}(x-s)+\frac{1}{s}$

$=-\frac{1}{s^2}x+\frac{2}{s}$

接線: $y=-\frac{1}{(t-1)^2}(x-t)-\frac{t-2}{t-1}$

$=-\frac{1}{(t-1)^2}x+\frac{t-(t-2)(t-1)}{(t-1)^2}$

傾き比較で $s^2=(t-1)^2$

$\therefore s=\pm(t-1)$

(i) $s=t-1$のとき

$\frac{2}{s}=\frac{t-(t-2)(t-1)}{(t-1)^2}$

$\Leftrightarrow \frac{2}{s}=\frac{s+1-(s-1)s}{s^2}$

$\Leftrightarrow 2s=-s^2+2s+1$

$\therefore s^2=1 \quad \therefore s=1 \quad (\because s>0)$

接線は $y=-x+2$

(ii) $s=1-t$のとき

$\frac{2}{s}=\frac{t-(t-2)(t-1)}{(t-1)^2}$

$\Leftrightarrow \frac{2}{s}=\frac{1-s-(-1-s)(-s)}{s^2}$

$\Leftrightarrow 2s=1-s-(s^2+s)$

$\Leftrightarrow s^2+4s-1=0$

$\therefore s=-2+\sqrt{5} \quad (\because s>0)$

接線は $y=-\left(\frac{1}{\sqrt{5}-2}\right)^2x+\frac{2}{\sqrt{5}-2}$

$=-(\sqrt{5}+2)^2x+2\sqrt{5}+4$

$=-(9+4\sqrt{5})x+2\sqrt{5}+4$

$\therefore \ l_1: \underline{y=-x+2}$ //

$l_2: \underline{y=-(9+4\sqrt{5})x+2\sqrt{5}+4}$ //

(2) C_1とC_2の交点

$\frac{1}{x}=\frac{2-x}{x-1}$

$\Leftrightarrow x-1=-x^2+2x$

$\Leftrightarrow x^2-x-1=0$

$\therefore x=\frac{1+\sqrt{5}}{2} \quad (\because x>0)$

交点 $\left(\dfrac{1+\sqrt{5}}{2}, \dfrac{\sqrt{5}-1}{2}\right)$

これを通り ℓ_2 に垂直な直線は

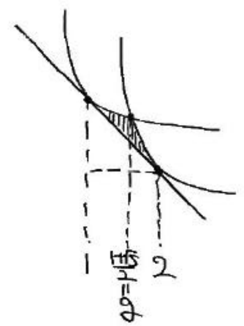
$$y=(\sqrt{5}-2)^2\left(x-\frac{1+\sqrt{5}}{2}\right)+\frac{\sqrt{5}-1}{2}$$

$$=(9-4\sqrt{5})x+\frac{(9-4\sqrt{5})(-1+\sqrt{5})+\sqrt{5}-1}{2}$$

$$=(9-4\sqrt{5})x+5-2\sqrt{5}$$

C_2 と ℓ_1 の接点は $(2, 0)$

(3)



$\dfrac{1+\sqrt{5}}{2}=a$ とおく

求める面積は

$$\int_1^a \frac{1}{x}dx+\int_a^2\left(\frac{1}{x-1}-1\right)dx-1\cdot1\cdot\frac{1}{2}$$

$$=\Big[\log|x|\Big]_1^a+\Big[\log|x-1|-x\Big]_a^2-\frac{1}{2}$$

$$=\log a-2-\log(a-1)+a-\frac{1}{2}$$

$$=\log\frac{1+\sqrt{5}}{2}-\log\frac{\sqrt{5}-1}{2}+\frac{1+\sqrt{5}}{2}-\frac{5}{2}$$

$$=\log\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-4}{2}$$

$$=\log\frac{6+2\sqrt{5}}{4}+\frac{\sqrt{5}}{2}-2$$

$$=\log\frac{3+\sqrt{5}}{2}+\frac{\sqrt{5}}{2}-2$$

$\boxed{3}$

(1) 確率 P で 2倍

$1-P$ で $\sqrt{2}$ 倍かつ $\frac{\pi}{4}$ 回転

$(1+i)^4=-4$ より $A(-2)$ は不可能

$A(-4)$ は可能、

$A(2i)$ は可能

$(1+i)^6=-8i$ より $A(-4i)$ は不可能、

$A(2+2i)$ は可能、$A(2-2i)$ は不可能、

ア…$A(-2), A(-4i), A(2-2i)$

(2)

$$P(t=2 \text{ で } A(2+2i))$$

$$={}_2C_1\cdot P(1-P)=2P(1-P)$$

(3) $P(t=5 \text{ で } A(-8))$

$$={}_5C_4(1-P)^4P=5P(1-P)^4$$

(4)

$R(8)$

$$=P^8+(1-P)^8$$

$R(9)$

$$=P^9+{}_9C_1P(1-P)^8$$

$$=P^9+9P(1-P)^8$$

$R(16)$

$$=P^{16}+{}_{16}C_8P^8(1-P)^8+(1-P)^{16}$$

$$=P^{16}+12870P^8(1-P)^8+(1-P)^{16}$$

(5)

最大は 2の2019乗

$$\begin{array}{r} 252 \\ 8\,\overline{)2019} \\ \underline{16} \\ 41 \\ \underline{40} \\ 19 \\ \underline{16} \\ 3 \end{array}$$

つまり 252回転と 2^3 倍で

大きさは

$$(\sqrt{2})^{2016}\cdot 2^3=2^{1008}\cdot 2^3$$

$$=2^{1011}$$

最小は 2の1011乗