2019 獨協医科 (医)

$\boxed{1}$ (1)

(i)

$(2^x)^2 - 8\cdot 2^x + 3^{\log_3 16} - 4 = 0$ ($3^{\log_3 16} = 16$)

$\Leftrightarrow \underline{t^2 - 8t + 12 = 0}$ $\swarrow 2^x = t$

$\therefore t = \underline{2, 6}$

$\therefore x = \underline{1}, \underline{\log_2 6}$

(ii) 真数条件 $3<x<5$

$a>1$ のとき

$\log_a (x-3) - \dfrac{\log_a(5-x)}{\log_a a^2} < \log_a \sqrt{x}$

$\Leftrightarrow \log_a \dfrac{x-3}{\sqrt{5-x}} < \log_a \sqrt{x}$

$\Leftrightarrow x-3 < \sqrt{5x-x^2}$

$\downarrow$ $(3<x<5)$

$x^2 - 6x + 9 < 5x - x^2$

$\Leftrightarrow 2x^2 - 11x + 9 < 0$

$\Leftrightarrow (2x-9)(x-1) < 0$

$\underline{3 < x < \frac{9}{2}}$

$0<a<1$ のとき

$x-3 > \sqrt{5x-x^2}$

$\downarrow$

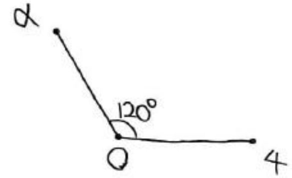
$(2x-9)(x-1)>0$

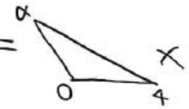
$\therefore \underline{\frac{9}{2} < x < 5}$

(2)

$\alpha = \underline{4}\left(\cos \underline{\frac{2}{3}\pi} + i\sin\frac{2}{3}\pi\right)$

$\beta = \underline{\frac{1}{2}}\left(\cos \underline{\frac{1}{3}\pi} + i\sin\frac{1}{3}\pi\right)$



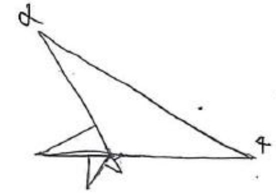
$D_n =$  $\times \left(\frac{1}{4}\right)^{n-1}$

$= \frac{1}{2}\cdot 4 \cdot 4 \sin 120° \times \left(\frac{1}{2}\right)^{2n-2}$

$= 4\sqrt{3}\left(\frac{1}{2}\right)^{2n-2}$

$= \sqrt{3}\left(\frac{1}{2}\right)^{2n-4}$

$\therefore D_2 = \underline{\sqrt{3}}$



$S_2 = D_1 + \frac{1}{2}D_2$

$= 4\sqrt{3} + \frac{\sqrt{3}}{2}$

$= \underline{\frac{9}{2}\sqrt{3}}$

$S_6 = S_5$ $\quad \min n = \underline{6}$

$\boxed{2}$

(1)

$a_1 = \frac{3}{6} = \underline{\frac{1}{2}}$

n回目 $\quad$ n+1回目

①,⑤,③ すべて同じ $\quad a_n \xrightarrow{\frac{1}{2}} a_{n+1}$

①,⑤,③ すべて同じではない) $\quad 1-a_n \nearrow \frac{1}{6}$

$a_{n+1} = \frac{1}{2}a_n + \frac{1}{6}(1-a_n)$

$= \underline{\frac{1}{3}}a_n + \underline{\frac{1}{6}}$

$a_2 = \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{6} = \underline{\frac{1}{3}}$

$a_n = ?\left(1 + \frac{1}{3^{n-1}}\right)$

$\downarrow$ $a_1 = \frac{1}{2}$ より $? = \frac{1}{4}$

$a_n = \underline{\frac{1}{4}}\left(1 + \underline{\frac{1}{3^{n-1}}}\right)$

(2)

n $\quad$ n+1

すべて同色 $\quad p_n \xrightarrow{\frac{1}{6}} p_{n+1}$

黒が1つか4つ $\quad q_n \xrightarrow{\frac{1}{6}} q_{n+1}$ $\quad \frac{5}{6}$, $\frac{1}{6}$

黒が2つか3つ $\quad r_n \nearrow \frac{2}{6}$

$p_{n+1} = \underline{\frac{1}{6}}p_n + \underline{\frac{1}{6}}q_n$

$q_{n+1} = \underline{\frac{5}{6}}p_n + \underline{\frac{1}{6}}q_n + \underline{\frac{1}{3}}r_n$

p_4

$= \frac{1}{6}p_3 + \frac{1}{6}q_3$

$= \frac{1}{6}\left(p_2 + \frac{1}{3}q_2 + \frac{1}{3}r_2\right)$

$= \frac{1}{6}\left\{p_2 + \frac{1}{3}(1-p_2)\right\}$

$= \frac{1}{6}\left(\frac{2}{3}p_2 + \frac{1}{3}\right)$

$= \frac{1}{9}\left(\frac{1}{6}p_1 + \frac{1}{6}q_1\right) + \frac{1}{18}$

$= \frac{1}{54}\underbrace{(p_1+q_1)}_{1} + \frac{1}{18} = \underline{\frac{2}{27}}$

$a_4 = \frac{7}{27}$ より $\quad b_4 = 2a_4 - p_4$

$= \frac{12}{27} = \underline{\frac{4}{9}}$

$\boxed{3}$

(1)
$\overrightarrow{OP}=\overrightarrow{OA}+p\overrightarrow{AB}=\begin{pmatrix}4\\-5+3p\\2+3p\end{pmatrix}$

$\overrightarrow{OQ}=\overrightarrow{OC}+q\overrightarrow{CD}=\begin{pmatrix}-1+2q\\-1+2q\\4+q\end{pmatrix}$

$\downarrow$ O.P.Q 一直線上

$k\overrightarrow{OP}=\overrightarrow{OQ}$

$\Leftrightarrow \begin{cases}4k=-1+2q\\(-5+3p)k=-1+2q\\(2+3p)k=4+q\end{cases}$

$\therefore \underline{p=3}$ $k=\frac{1}{2}$ $\underline{q=\frac{3}{2}}$

(2)
$\overrightarrow{PQ}\cdot\overrightarrow{AB}$

$=\begin{pmatrix}-5+2q\\4+2q-3p\\2+q-3p\end{pmatrix}\cdot\begin{pmatrix}0\\3\\3\end{pmatrix}$

$=18+9q-18p=0$

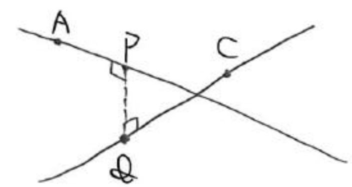
$\Leftrightarrow \underline{2-2p+q=0}$ $\cdots$ ①

$\overrightarrow{PQ}\cdot\overrightarrow{CD}$

$=\overrightarrow{PQ}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix}$

$=9q-9p=0$ $\cdots$ ②

$PQ\perp AB$ かつ $PQ\perp CD$ で線分 PQ は最小.



①, ②より $p=q=2$

$\therefore \underline{P(4,1,8),\ Q(3,3,6)}$

$\overrightarrow{PA_1}=\begin{pmatrix}0\\-6\\-6\end{pmatrix}$ $\overrightarrow{Q_1C}=\begin{pmatrix}-4\\-4\\-2\end{pmatrix}$

$\underbrace{\overrightarrow{PA_1}\cdot\overrightarrow{Q_1C}}_{36}=\underbrace{|\overrightarrow{PA_1}|}_{6\sqrt{2}}\underbrace{|\overrightarrow{Q_1C}|}_{6}\cos\alpha$

$\cos\alpha=\frac{1}{\sqrt{2}}$

$\therefore \alpha=\theta=\underline{\frac{1}{4}\pi}$

$\boxed{4}$

微分可能なら連続なので

$x=-1$ で $2+a=-2b+2c+1$

$x=2$ で $b+2c-2=16b-c-5d+2$

微分係数を考えて

$x=-1$ で $-2=(2bx-1)|_{x=-1}$
$=-2b-1$

$\therefore \underline{b=\frac{1}{2}}$

$x=2$ で $(2bx-1)|_{x=2}=[(3-3c)x^2+8bx-3]|_{x=2}$

$4b-1=12-12c+16b-3$

$\Leftrightarrow 1=17-12c$

$\therefore c=\frac{16}{12}=\underline{\frac{4}{3}}$

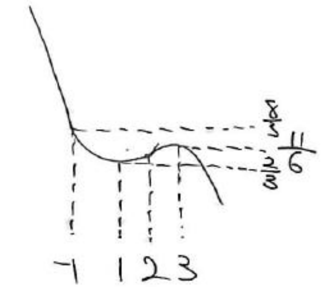
$2+a=-1+\frac{8}{3}+1$ $\therefore \underline{a=\frac{2}{3}}$

$\frac{1}{2}+\frac{8}{3}-2=8-\frac{4}{3}-5d+2$

$\underline{d=\frac{3}{2}}$

(1)
$f(x)=\begin{cases}-2x+\frac{2}{3} & (x<-1)\\ \frac{1}{2}x^2-x+\frac{7}{6} & (-1\leqq x<2)\\ -\frac{1}{3}x^3+2x^2-3x+\frac{11}{6} & (x\geqq 2)\end{cases}$

$f'(x)=\begin{cases}-2 & (x<-1)\\ x-1 & (-1\leqq x<2)\\ -x^2+4x-3 & (x\geqq 2)\end{cases}$



$f(x)=k$ で k が

$\underline{\frac{2}{3}<k<\frac{11}{6}}$

なら異なる3つの実数解をもつ.

$\frac{1}{2}x^2-x+\frac{7}{6}=\frac{11}{6}$

$\Leftrightarrow x^2-2x-\frac{4}{3}=0$

$\Leftrightarrow x=1\pm\sqrt{\frac{7}{3}}$

α の範囲は

$\underline{\frac{3-\sqrt{21}}{3}<\alpha<1}$

$P(2,\frac{7}{6})$ での接線は

$y=-(x-2)+\frac{7}{3}$
$=-x+\frac{13}{3}$

これと $y=-2x+\frac{2}{3}$ を連立して

$x=-\frac{11}{3}$

$f'(x)=-x^2+4x-3=-1$

$\Leftrightarrow 0=x^2-4x+2$

$\therefore x=2+\sqrt{2}$ $(2<x<4)$

このときのy座標は

$$\begin{array}{r|rrrr}
 & & & -\frac{1}{3} & \frac{2}{3} \\ \hline
1\ -4\ 2 & -\frac{1}{3} & 2 & -3 & \frac{11}{6} \\
 & -\frac{1}{3} & \frac{4}{3} & -\frac{2}{3} & \\ \hline
 & & \frac{2}{3} & -\frac{7}{3} & \frac{11}{6} \\
 & & \frac{2}{3} & -\frac{8}{3} & \frac{4}{3} \\ \hline
 & & & \frac{1}{3} & \frac{1}{2}
\end{array}$$

$-\frac{1}{3}x^3+2x^2-3x+\frac{11}{6}$

$=(x^2-4x+2)\left(-\frac{1}{3}x+\frac{2}{3}\right)$

$\quad +\frac{1}{3}x+\frac{1}{2}$

$\downarrow$ $x=2+\sqrt{2}$ を代入

$\frac{2+\sqrt{2}}{3}+\frac{1}{2}=\frac{7+2\sqrt{2}}{6}$

$\therefore R\left(2+\sqrt{2},\ \frac{7+2\sqrt{2}}{6}\right)$

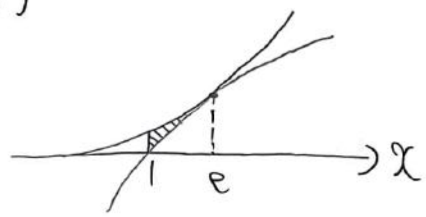
$\boxed{5}$

接点のx座標をtとする.

$\begin{cases} 傾き一致：2at=\frac{2}{t} \Leftrightarrow at^2=1 \\ y座標一致：at^2+1=2\log t \end{cases}$

$\therefore t=e,\ a=\frac{1}{e^2}$

(1)



T

$=\int_1^e\left(\frac{1}{e^2}x^2+1-2\log x\right)dx$

$=\left[\frac{1}{3e^2}x^3+x-2x\log x+2x\right]_1^e$

$=\frac{1}{3}e+e-\frac{1}{3e^2}-3$

$=\frac{4}{3}e-\frac{1}{3e^2}-3$

V_1

$=\int_1^e 2\pi x\left(\frac{1}{e^2}x^2+1-2\log x\right)dx$

$=2\pi\int_1^e\left(\frac{1}{e^2}x^3+x-2x\log x\right)dx$

$=2\pi\left[\frac{1}{4e^2}x^4+\frac{x^2}{2}\right.$

$\qquad\left.-x^2\log x+\frac{x^2}{2}\right]_1^e$

$=2\pi\left\{\frac{e^2}{4}+\frac{e^2}{2}-e^2+\frac{e^2}{2}-\frac{1}{4e^2}-1\right\}$

$=2\pi\left(\frac{e^2}{4}-\frac{1}{4e^2}-1\right)$

$=\frac{1}{2}\left(e^2-\frac{1}{e^2}-4\right)\pi$

(2)

L

$=\int_{\sqrt{5}}^{2\sqrt{3}}\sqrt{1+\left(\frac{2}{x}\right)^2}\,dx$

$=\int_{\sqrt{5}}^{2\sqrt{3}}\frac{\sqrt{x^2+4}}{x}\,dx$

$\downarrow$ $\sqrt{x^2+4}=t$
$x^2+4=t^2$
$2x\,dx=2t\,dt$

$=\int_3^4\frac{t}{t^2-4}\cdot t\,dt$

$=\int_3^4\frac{t^2-4+4}{t^2-4}\,dt$

$=\int_3^4\left\{1+\frac{4}{(t-2)(t+2)}\right\}dt$

$=\int_3^4\left\{1+\frac{1}{t-2}-\frac{1}{t+2}\right\}dt$

$=\Big[t+\log|t-2|-\log|t+2|\Big]_3^4$

$=4+\log 2-\log 6$

$\quad -(3-\log 5)$

$=1+\log\frac{5}{3}$

※ V_1 はバウムクーヘン積分を使っています.