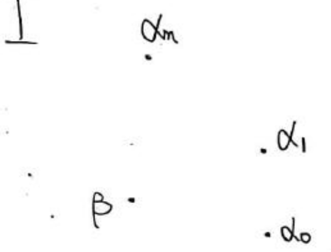
2019 中央大(理工)

I



$$\alpha_m-\beta=(\alpha_0-\beta)\left(\cos\frac{m}{n}2\pi+i\sin\frac{m}{n}2\pi\right)$$

$$\Leftrightarrow 2i=\beta-\beta\left(\cos\frac{2m\pi}{n}+i\sin\frac{2m\pi}{n}\right)$$

$$\downarrow$$

$$\beta=\frac{2i}{1-\cos\frac{2m\pi}{n}-i\sin\frac{2m\pi}{n}}$$

$$=\frac{2i\left(1-\cos\frac{2m\pi}{n}+i\sin\frac{2m\pi}{n}\right)}{\left(1-\cos\frac{2m\pi}{n}\right)^2+\sin^2\frac{2m\pi}{n}}$$

$$=\frac{i\left(1-\cos\frac{2m\pi}{n}+i\sin\frac{2m\pi}{n}\right)}{1-\cos\frac{2m\pi}{n}}$$

$$=\frac{-\sin\frac{2m\pi}{n}}{1-\left(1-2\sin^2\frac{m\pi}{n}\right)}+i$$

$$=\frac{-2\sin\frac{m\pi}{n}\cos\frac{m\pi}{n}}{2\sin^2\frac{m\pi}{n}}+i$$

$$=-\frac{1}{\tan\frac{m\pi}{n}}+i$$

$$=\underline{\tan\left(\frac{m}{n}-\frac{1}{2}\right)\pi+i}_{/\!/}$$

ア…ⓐ

イ…ⓑ

$$P_0Q^2=|\beta|^2$$

$$=\tan^2\left(\frac{m}{n}-\frac{1}{2}\right)\pi+1$$

$$=\frac{1}{\cos^2\left(\frac{m}{n}-\frac{1}{2}\right)\pi}$$

$$=\frac{1}{\sin^2\frac{m}{n}\pi}$$

$$\therefore P_0Q=\underline{\frac{1}{\sin\frac{m}{n}\pi}}_{/\!/}$$

ウ…ⓘ

また

$$\alpha_k-\beta$$

$$=(\alpha_0-\beta)\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

$$\downarrow$$

$$\alpha_k$$

$$=\beta-\beta\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

$$=\beta-\left(-\frac{1}{\tan\frac{m\pi}{n}}+i\right)\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

$$=\beta-P_0Q\left(-\cos\frac{m\pi}{n}+i\sin\frac{m}{n}\pi\right)\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

$$=\beta+P_0Q\left\{\cos\left(-\frac{m\pi}{n}\right)+i\sin\left(-\frac{m\pi}{n}\right)\right\}\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$$

$$=\beta+P_0Q\left\{\cos\underline{(2k-m)\frac{\pi}{n}}_{/\!/}+i\sin(2k-m)\frac{\pi}{n}\right\}$$

エ…ⓕ

II

$$a(x+1)+2=-\frac{1}{x}$$

$$\Leftrightarrow ax^2+(a+2)x+1=0$$

解と係数の関係より

$$\alpha+\beta=\underline{-\frac{a+2}{a}}_{/\!/},\quad \alpha\beta=\underline{\frac{1}{a}}_{/\!/}$$

オ…ⓑ　カ…ⓒ

$$S(a)$$

$$=\int_\alpha^\beta\left\{ax+(a+2)+\frac{1}{x}\right\}$$

$$=\left[\frac{a}{2}x^2+(a+2)x+\log|x|\right]_\alpha^\beta$$

$$=\frac{a}{2}(\beta^2-\alpha^2)+(a+2)(\beta-\alpha)+\underbrace{\log\frac{|\beta|}{|\alpha|}}$$

$$\log\frac{-\beta}{-\alpha}=\log\frac{\beta}{\alpha}$$

キ…ⓓ

$$S'(a)$$

$$=\frac{1}{2}(\beta^2-\alpha^2)+\frac{a}{2}\left(2\beta\frac{d\beta}{da}-2\alpha\frac{d\alpha}{da}\right)$$

$$+(\beta-\alpha)+(a+2)\left(\frac{d\beta}{da}-\frac{d\alpha}{da}\right)$$

$$+\frac{1}{\beta}\cdot\frac{d\beta}{da}-\frac{1}{\alpha}\cdot\frac{d\alpha}{da}$$

$$=\frac{1}{2}(\beta^2-\alpha^2)+\beta-\alpha$$

$$+\left(a\beta+(a+2)+\frac{1}{\beta}\right)\frac{d\beta}{da}$$

$$-\left(a\alpha+(a+2)+\frac{1}{\alpha}\right)\frac{d\alpha}{da}$$

$$=\frac{1}{2}(\beta-\alpha)(\beta+\alpha)+\beta-\alpha$$

$$=\underline{\frac{1}{2}(\beta-\alpha)(\alpha+\beta+2)}_{/\!/}$$

ク…ⓙ

$$S'(a)=0$$

$$\Leftrightarrow \alpha+\beta+2=-\frac{2}{a}+1=0$$

$$\Leftrightarrow \underline{a=2}_{/\!/}$$

ケ…ⓔ

$$(\beta-\alpha)^2=(\alpha+\beta)^2-4\alpha\beta$$

$$=\frac{a^2+4a+4}{a^2}-\frac{4}{a}$$

$$=\frac{a^2+4}{a^2}$$

$$\therefore \beta-\alpha=\frac{\sqrt{a^2+4}}{a}$$

$S(a)$

$= \frac{a}{2}\left(-\frac{a+2}{a}\right)\frac{\sqrt{a^2+4}}{a}$

$+ (a+2)\frac{\sqrt{a^2+4}}{a} + \log\frac{\beta}{\alpha}$

$= \frac{a+2}{2a}\sqrt{a^2+4} + \log\frac{-(a+2)+\sqrt{a^2+4}}{-(a+2)-\sqrt{a^2+4}}$

$= \frac{a+2}{2a}\sqrt{a^2+4} + \log\frac{a+2-\sqrt{a^2+4}}{a+2+\sqrt{a^2+4}}$ //

コ…② //　サ…ⓟ //

$a=2$で最小をとるのでその値は

$S(2)$

$= \sqrt{8} + \log\frac{4-\sqrt{8}}{4+\sqrt{8}}$

$= 2\sqrt{2} + \log\frac{2-\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= 2\sqrt{2} + \log(3-2\sqrt{2})$

$= 2\sqrt{2} + 2\log(\sqrt{2}-1)$

シ…ⓗ //

III

(1)

R_5が$(-1,0)$であるには

5回中1回$\frac{\pi}{3}$, 4回$\frac{\pi}{6}$移動.

${}_5C_1 \frac{1}{2}\left(\frac{1}{2}\right)^4 = \frac{5}{32}$ //

(2)

R_9がx軸上にあるには

総移動が2πまたは3π.

${}_9C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^9$

$= \frac{3\cdot4\cdot7+1}{512} = \frac{85}{512}$ //

(3)

n秒後　n+1秒後

x軸またはy軸上　$P_n \xrightarrow{0} P_{n+1}$

それ以外　$1-P_n \xrightarrow{\frac{1}{2}} P_{n+1}$

$P_{n+1} = \frac{1}{2}(1-P_n)$ //

(4)

$P_{n+1} - \frac{1}{3} = \frac{-1}{2}\left(P_n - \frac{1}{3}\right)$

$\{P_n - \frac{1}{3}\}$の一般項は

$P_n - \frac{1}{3} = (\underbrace{P_1}_{0} - \frac{1}{3})\left(-\frac{1}{2}\right)^{n-1}$

$\therefore P_n = \frac{1}{3} - \frac{1}{3}\left(-\frac{1}{2}\right)^{n-1}$ //