# 2018 獨協医科（医）

## $\boxed{1}$

(1) $\arg\alpha^3 = \frac{\pi}{3}$

$\arg\beta^3 = \frac{\pi}{4}$

↓

$\arg\alpha^3\beta^3 = \frac{\pi}{3} + \frac{\pi}{4} = \underline{\frac{7}{12}\pi}$ //

$\arg\frac{\alpha^3}{\beta^3} = \frac{\pi}{3} - \frac{\pi}{4} = \underline{\frac{\pi}{12}}$ //

$z = \cos\frac{\pi}{36} + i\sin\frac{\pi}{36}$

$1 + z + \cdots + z^{n-1}$

$= \frac{1-z^n}{1-z} = 0$

これを満たす最小の $n$ は $\underline{72}$ //

$z^{96} = \cos\frac{8}{3}\pi + i\sin\frac{8}{3}\pi$

$= \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$

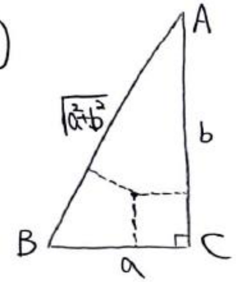
$2z^{96} = -1 + \sqrt{3}i$

$2z^{96} + \frac{1}{2z^{96}}$

$= -1 + \sqrt{3}i + \frac{1}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= \underline{-\frac{5}{4} + \frac{3\sqrt{3}}{4}i}$ //

(2)



$a \cdot b \cdot \frac{1}{2} = \frac{r}{2}\left(a + b + \sqrt{a^2+b^2}\right)$

$\Leftrightarrow \frac{ab}{a+b+\sqrt{a^2+b^2}} = r$

$\Leftrightarrow \frac{ab\left(a+b-\sqrt{a^2+b^2}\right)}{2ab} = r$

$\therefore a + b - \underline{2r} = \sqrt{a^2+b^2}$ //

$r = 2$ のとき

$a + b - 4 = \sqrt{a^2+b^2}$

2乗して

$16 - 8a - 8b + 2ab = 0$

$\Leftrightarrow ab - 4a - 4b + 8 = 0$

$\Leftrightarrow (a-4)(b-4) = 8$

$(a, b) = \underline{(8, 6), (12, 5)}$ //

$r = 2^{100}$ のとき

$2^{202} - 2^{102}a - 2^{102}b + 2ab = 0$

$\Leftrightarrow ab - 2^{101}a - 2^{101}b + 2^{201} = 0$

$\Leftrightarrow (a - 2^{101})(b - 2^{101}) = 2^{201}$

| | |
|---|---|
| $2^{201}$ | $1$ |
| $2^{200}$ | $2$ |
| $\vdots$ | |
| $2^{101}$ | $2^{100}$ |

全部で $\underline{101 組}$ //

$r = p^{100}$ のとき

$4p^{200} - 4p^{100}a - 4p^{100}b + 2ab = 0$

$\Leftrightarrow ab - 2p^{100}a - 2p^{100}b + 2p^{200} = 0$

$\Leftrightarrow (a - 2p^{100})(b - 2p^{100}) = 2p^{200}$

$a \geqq b$ 不向ならば $2 \times 201 = 402$

組なので 求める答えは $\underline{201 組}$ //

## $\boxed{2}$

(1)

P(赤をとる)

$= \frac{30}{150} \cdot \frac{7}{9} + \frac{120}{150} \cdot \frac{2}{9}$

$= \frac{1}{5} \cdot \frac{7}{9} + \frac{4}{5} \cdot \frac{2}{9} = \frac{15}{45} = \frac{1}{3}$

P(白をとる) $= 1 - \frac{1}{3} = \frac{2}{3}$.

P(赤を2回とる)

$= {}_6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 = \underline{\frac{80}{243}}$ //

P(白を連続して取り出さない)

$=$ P(オール赤)

$+$ P(赤5, 白1)

$+$ P(赤4, 白2)

$+$ P(赤3, 白3)

$= \left(\frac{1}{3}\right)^6 + {}_6C_1 \left(\frac{1}{3}\right)^5 \frac{2}{3}$

$+ {}_5C_2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2$ ← VRVRVRVRV

$+ {}_4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3$ VRVRVRV

$= \underline{\frac{85}{729}}$ //

(2)

P(赤を5回以上連続で出す)

$= \left(\frac{1}{3}\right)^{10} + 2\left(\frac{1}{3}\right)^9 \frac{2}{3}$

$+ 2\left(\frac{1}{3}\right)^8 \frac{2}{3} + \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2$ ? W R~R / W R~R W / R~R W ?

$+ 2\left(\frac{1}{3}\right)^7 \frac{2}{3}$ ?? W R~R / ?W R~R W

$+ 2\left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^2$ W R~R W? / R~R W ??

$+ 2\left(\frac{1}{3}\right)^6 \frac{2}{3}$ ??? W R~R / ??W R~R W

$+ 3\left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^2$ ?W R~R W ? / W R~R W ?? / R~R W ???

$+2\left(\frac{1}{3}\right)^5\frac{2}{3}+4\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2$

$=\frac{13}{729}$

$P$(4回目白∩赤を5回以上連続)

$=2\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2$ ???WWR~R

???WR~RW

$+\left(\frac{1}{3}\right)^6\frac{2}{3}$ ???WR~R

$=\frac{10}{3^7}$

$P_{4回目白}$(赤を5回以上連続)

$=\dfrac{\frac{10}{3^7}}{\frac{13}{729}}=\frac{10}{39}$

(3)

$P$(赤をとる)

$=\frac{m}{150}\cdot\frac{7}{9}+\frac{150-m}{150}\cdot\frac{2}{7}$

$=\frac{300+5m}{1350}=\frac{m+60}{270}=p$ とおく

$P_k={}_{100}C_k\,p^k(1-p)^{100-k}$

$\frac{P_{k+1}}{P_k}=\dfrac{\frac{100!}{(k+1)!(99-k)!}}{\frac{100!}{k!(100-k)!}}\cdot\frac{p}{1-p}$

$=\frac{100-k}{k+1}\cdot\frac{p}{1-p}>1$

$\Leftrightarrow 100p-kp>k+1-kp-p$

$\Leftrightarrow p>\frac{k+1}{101}$

$\frac{P_{70}}{P_{69}}>1$ より

$p>\frac{71}{101}$

$\frac{P_{71}}{P_{70}}<1$ より

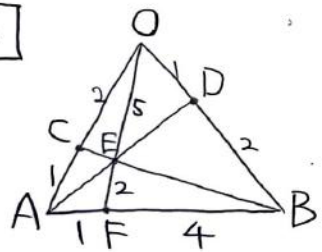
$p<\frac{72}{101}$

$\Downarrow$

$\frac{71}{101}<\frac{m+60}{270}<\frac{72}{101}$

$\therefore m=128,\ 129$

$\boxed{3}$




$\overrightarrow{OE}=\frac{5}{7}\left(\frac{4}{5}\overrightarrow{OA}+\frac{1}{5}\overrightarrow{OB}\right)$

$=\frac{4}{7}\overrightarrow{OA}+\frac{1}{7}\overrightarrow{OB}$

$\overrightarrow{OF}=\frac{4}{5}\overrightarrow{OA}+\frac{1}{5}\overrightarrow{OB}$

$\triangle OAB=1$ とする

$\triangle CFD=1-\frac{1}{3}\cdot\frac{2}{3}-\frac{1}{3}\cdot\frac{1}{5}-\frac{4}{5}\cdot\frac{2}{3}$

$=\frac{8}{45}=\frac{\triangle CFD}{\triangle OAB}$

$\triangle OAB=10\sqrt{3}$ のとき

$\triangle CFD=\frac{8}{45}\cdot10\sqrt{3}$

$=\frac{16}{9}\sqrt{3}=\frac{1}{2}(-3\Omega)^2\sin60^\circ$

$\Leftrightarrow \frac{16}{9}=\frac{1}{4}(-3\Omega)^2$

$\therefore (-3\Omega)^2=\frac{64}{9}$

$\overrightarrow{CD}\cdot\overrightarrow{CF}=(-3\Omega)^2\cos60^\circ$

$=\frac{32}{9}$

$\overrightarrow{CD}=\vec{\alpha}$ $\overrightarrow{CF}=\vec{\beta}$ とする.

$|\vec{\alpha}|=|\vec{\beta}|=\frac{8}{3}$ $\vec{\alpha}\cdot\vec{\beta}=\frac{32}{9}$

$\vec{\alpha}=\overrightarrow{OD}-\overrightarrow{OC}$

$=\frac{1}{3}\vec{b}-\frac{2}{3}\vec{a}$

$\vec{\beta}=\overrightarrow{OF}-\overrightarrow{OC}$

$=\frac{2}{15}\vec{a}+\frac{1}{5}\vec{b}$

$\vec{a},\vec{b}$ について解くと

$\vec{a}=\frac{3}{8}(-3\vec{\alpha}+5\vec{\beta})$

$\vec{b}=\frac{3}{4}(\vec{\alpha}+5\vec{\beta})$

$|\vec{a}|^2=\frac{9}{64}|-3\vec{\alpha}+5\vec{\beta}|^2$

$=\cdots$

$=19$

$|\vec{b}|^2=\frac{9}{16}|\vec{\alpha}+5\vec{\beta}|^2$

$=\cdots$

$=124$

$\therefore |\overrightarrow{OA}|=|\vec{a}|=\sqrt{19}$

$|\overrightarrow{OB}|=|\vec{b}|=2\sqrt{31}$

$\boxed{4}$

$C_1=\int_0^{\pi}\cos^2t\,dt$

$=\int_0^{\pi}\frac{1+\cos2t}{2}dt$

$=\left[\frac{t}{2}+\frac{1}{4}\sin2t\right]_0^{\pi}$

$=\frac{1}{2}\pi$

$a_{n+1}$

$= \int_0^{\pi} (\cos t + a_n t)\cos t\,dt$

$= \int_0^{\pi} \cos^2 t\,dt + a_n \int_0^{\pi} t\cos t\,dt$

$= \frac{1}{2}\pi + a_n \left[ t\sin t + \cos t \right]_0^{\pi}$

$= -2a_n + \frac{1}{2}\pi$ //

$\Leftrightarrow a_{n+1} - \frac{\pi}{6} = -2\left(a_n - \frac{\pi}{6}\right)$

$\Downarrow$

$a_n - \frac{\pi}{6} = \left(a_1 - \frac{\pi}{6}\right)(-2)^{n-1}$

$\therefore a_n = \left\{ \frac{(-2)^{n-1}}{3} + \frac{1}{6} \right\}\pi$ //

$\sum_{n=1}^{\infty} \left\{ \frac{1}{k^n} \cdot \frac{(-2)^{n-1}}{3}\pi \right\}$

$= \sum_{n=1}^{\infty} \left\{ -\frac{\pi}{6}\left(\frac{-2}{k}\right)^n \right\}$

収束するには $\left|\frac{-2}{k}\right| < 1$

解くと $k > 2$

$\therefore \min k = 3$ //

$S_k$

$= \sum_{n=1}^{\infty} \frac{1}{k^n} \left\{ \frac{(-2)^{n-1}}{3} + \frac{1}{6} \right\}\pi$

$= \sum_{n=1}^{\infty} \left\{ -\frac{1}{6}\left(\frac{-2}{k}\right)^n + \frac{1}{6}\left(\frac{1}{k}\right)^n \right\}\pi$

$= \left( \frac{\frac{1}{3k}}{1+\frac{2}{k}} + \frac{\frac{1}{6k}}{1-\frac{1}{k}} \right)\pi$

$= \left\{ \frac{1}{3(k+2)} + \frac{1}{6(k-1)} \right\}\pi$

$\sum_{k=3}^{\infty} \frac{6S_k}{\pi k}$

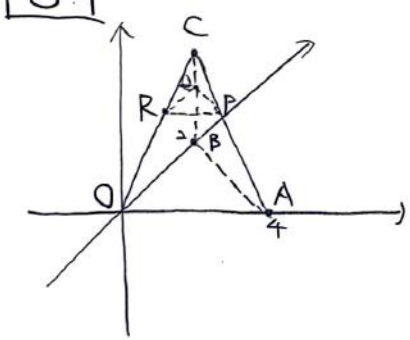
$= \sum_{k=3}^{\infty} \left\{ \frac{2}{k(k+2)} + \frac{1}{k(k-1)} \right\}$

$= \sum_{k=3}^{\infty} \left\{ \frac{1}{k} - \frac{1}{k+2} + \frac{1}{k-1} - \frac{1}{k} \right\}$

$= \sum_{k=3}^{\infty} \left( \frac{1}{k-1} - \frac{1}{k+2} \right)$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$ //

$\boxed{5}$




$P(4-2t, t, t)$

$Q(0, 2, t)$ $R(0, t, t)$

$HQ^2 = 4$ //

$HP^2 = (4-2t)^2 + t^2 = 5t^2 - 16t + 16$ //

$HP^2 - HQ^2$

$= 5t^2 - 16t + 12 = (5t-6)\underbrace{(t-2)}_{\leq 0}$

$0 \leq t < \frac{6}{5}$ のとき $HP > HQ$.

このとき

$S(t)$

$= (HP^2 - HR^2)\pi$

$= (4t^2 - 16t + 16)\pi$ //

$\frac{6}{5} \leq t \leq 2$ のとき

$S(t)$

$= (HQ^2 - HR^2)\pi$

$= (4 - t^2)\pi$

$= \pi(-t^2 + 4)$ //

$V$

$= \pi\int_0^{\frac{6}{5}} (4t^2 - 16t + 16)\,dt$

$+ \pi\int_{\frac{6}{5}}^{2} (4 - t^2)\,dt$

$= 4\pi\int_0^{\frac{6}{5}} (t-2)^2\,dt + \pi\left[4t - \frac{t^3}{3}\right]_{\frac{6}{5}}^{2}$

$= \cdots$

$= \frac{832}{75}\pi$ //