2017 東海大医2日目 (2/3)

1

(1)

(与式)

$= \lim_{t\to\infty}\left(\sqrt{9t^2-7t+5}-3t\right)$

$= \lim_{t\to\infty}\dfrac{-7t+5}{\sqrt{9t^2-7t+5}+3t} = -\dfrac{7}{6}$ //

(2)

1 □ 偶 … 4·3 = 12 とおり

2 □ 偶 … 4·2 = 8

3 □ 偶 …… 12

4 0 2 …… 1

4 1 偶 …… 2

4 2 0 … 1

4 3 0 … 1

+) 4 3 2 …… 1

38 個 //

(3)

$\tan(\alpha+\beta)$

$= \dfrac{4\sqrt{2}+2\sqrt{3}}{1-(\sqrt{2}+\sqrt{3})(3\sqrt{2}+\sqrt{3})}$

$= \dfrac{4\sqrt{2}+2\sqrt{3}}{-8-4\sqrt{6}}$

$= -\dfrac{2\sqrt{2}+\sqrt{3}}{4+2\sqrt{6}}$

$= -\dfrac{1}{2}\cdot\dfrac{2\sqrt{2}+\sqrt{3}}{\sqrt{6}+2}\cdot\dfrac{\sqrt{6}-2}{\sqrt{6}-2}$

$= -\dfrac{1}{4}(4\sqrt{3}-4\sqrt{2}+3\sqrt{2}-2\sqrt{3})$

$= \dfrac{1}{4}\sqrt{2} - \dfrac{1}{2}\sqrt{3}$ //

(4)

$7379 = 5311\cdot 1 + 2068$

$5311 = 2068\cdot 2 + 1175$ (4136)

$2068 = 1175\cdot 1 + 893$

$1175 = 893\cdot 1 + 282$

$893 = 282\cdot 3 + 47$ (846)

$\gcd(5311, 7379)$

$= \gcd(282, 47) = 47$ //

(5)

$\underbrace{|2(x-1)|}_{左} + \underbrace{|y-2|}_{右} \leq 4$

(左, 右) = (4, 0), (2, 2), (2, 1)

(2, 0), (0, 4) ~ (0, 1)

(0, 0)

$(x-1, y-2) = (\pm 2, 0), (\pm 1, \pm 2),$

$(\pm 1, \pm 1), (\pm 1, 0), (0, \pm 4)$

$\sim (0, \pm 1)$

$(0, 0)$ 計 21 個 //

(6)

$F(x)$

$= x^2\int_0^x \sin 5t\,dt + \int_0^x t\sin 5t\,dt$

$F'(x)$

$= 2x\int_0^x \sin 5t\,dt + x^2\sin 5x$

$+ x\sin 5x$

$F''(x)$

$= 2\int_0^x \sin 5t\,dt + 2x\sin 5x$

$+ 2x\sin 5x + 5x^2\cos 5x$

$+ \sin 5x + 5x\cos 5x$

$F''\left(\frac{\pi}{2}\right)$

$= 2\left[-\frac{1}{5}\cos 5t\right]_0^{\frac{\pi}{2}} + 4\cdot\frac{\pi}{2} + 1$

$= 2\pi + \dfrac{7}{5}$ //

(7)

$|\alpha|^2 = 1 \Leftrightarrow \alpha\bar{\alpha} = 1$

(左辺)

$= 27(1+\bar{\alpha}+(\bar{\alpha})^2+(\bar{\alpha})^3)$

$= 27\left(1+\dfrac{2-\sqrt{5}i}{3}+\dfrac{-1-4\sqrt{5}i}{9} + \dfrac{-22-7\sqrt{5}i}{27}\right)$

$= 27\cdot\dfrac{27+18-9\sqrt{5}i-3-12\sqrt{5}i-22-7\sqrt{5}i}{27}$

$= 20-28\sqrt{5}i$ //

2

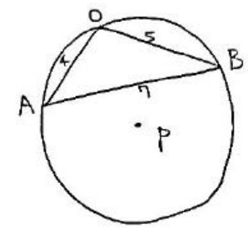
(1)

$|\overrightarrow{AB}|^2 = |\vec{b}-\vec{a}|^2$

$= \underbrace{|\vec{b}|^2}_{25} - 2\vec{a}\cdot\vec{b} + \underbrace{|\vec{a}|^2}_{16} = 49$

$\therefore\ \vec{a}\cdot\vec{b} = -4$ //

(2)



$|\overrightarrow{OP}-\vec{a}| = |\overrightarrow{OP}|$

$\Leftrightarrow |(s-1)\vec{a}+t\vec{b}|^2 = |s\vec{a}+t\vec{b}|^2$

$\Leftrightarrow (s-1)^2 16 - 8(s-1)t$

$= s^2 16 - 8st$

$\Leftrightarrow (-2s+1)16 + 8t = 0$

$\Leftrightarrow 4s - t - 2 = 0$ //

$|\overrightarrow{OP}-\vec{b}| = |\overrightarrow{OP}|$

$\Leftrightarrow |s\vec{a}+(t-1)\vec{b}|^2 = |s\vec{a}+t\vec{b}|^2$

$\Leftrightarrow (-2t+1)25 + 8s = 0$

$\Leftrightarrow 8S-50t+25=0$

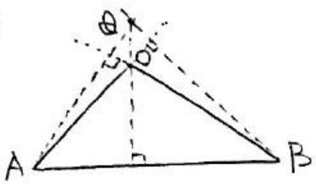
$\underline{-)\ 8S-2t-4=0}$

$-48t+29=0 \quad \therefore t=\frac{29}{48}$

$4S=t+2=\frac{125}{48}$

$\therefore S=\frac{125}{192}$

(3)



$\overrightarrow{OQ}=x\vec{a}+y\vec{b}$ とおく

$\overrightarrow{AQ}\cdot\vec{b}$

$=\{(x-1)\vec{a}+y\vec{b}\}\cdot\vec{b}$

$=-4(x-1)+25y=0$

$\overrightarrow{BQ}\cdot\vec{a}$

$=\{x\vec{a}+(y-1)\vec{b}\}\cdot\vec{a}$

$=16x-4(y-1)=0$

$\underline{+)\ -16x+16+100y=0}$

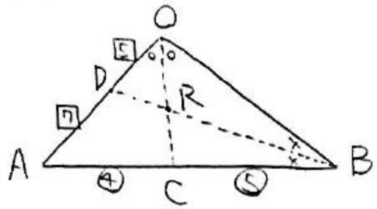
$96y+20=0$

$\therefore y=-\frac{5}{24}$

$4x=y-1=-\frac{29}{24} \quad x=-\frac{29}{96}$

$\therefore \overrightarrow{OQ}=-\frac{29}{96}\vec{a}-\frac{5}{24}\vec{b}$

(4)



$\overrightarrow{OC}=\frac{5\vec{a}+4\vec{b}}{4+5}=\frac{5}{9}\vec{a}+\frac{4}{9}\vec{b}$

$\overrightarrow{OD}=\frac{5}{5+7}\overrightarrow{OA}=\frac{5}{12}\vec{a}$

メネラウスの定理より

$\frac{9}{5}\cdot\frac{CR}{RO}\cdot\frac{5}{7}=1$

$\therefore CR:RO=7:9$

$\overrightarrow{OR}=\frac{9}{9+7}\overrightarrow{OC}$

$=\frac{5}{16}\vec{a}+\frac{4}{16}\vec{b}$

$=\frac{5}{16}\vec{a}+\frac{1}{4}\vec{b}$

$\boxed{3}$

(1) 連立

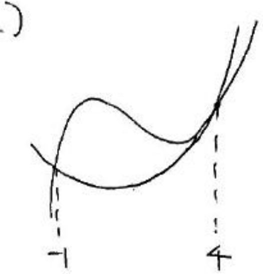
差$=x^3-7x^2+8x+16=0$

$\Leftrightarrow (x-4)(x^2-3x-4)=0$

$\Leftrightarrow (x-4)^2(x+1)=0$

$\therefore x=4,\ -1$

(2)



差の式より $x>-1$ で C が大きくて $x<-1$ で C が小さい．$x=4$ で重解なので上のようなグラフに．

(面積)

$=\int_{-1}^{4}$ 差 dx

$\frac{1}{12}$公式

$=\frac{1}{12}\{4-(-1)\}^4=\frac{625}{12}$

(3)

$f'(x)=3x^2-12x+8$

接線：$y=(3t^2-12t+8)(x-t)+t^3-6t^2+8t+16$

$=(3t^2-12t+8)x-2t^3+6t^2+16$

(4)

$D(t)$

$=(3t^2-12t+8)^2-4(2t^3-6t^2-16)$

$D'(t)$

$=2(3t^2-12t+8)(6t-12)-4(6t^2-12t)$

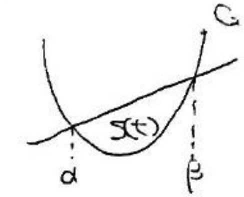
$=2(6t-12)(3t^2-12t+8-2t)$

$=12(t-2)(3t-2)(t-4)$

$\therefore t=\frac{2}{3},\ 2,\ 4$ で符号変わる．

ここで極値とる．

(5)



$S(t)=\frac{1}{6}(\beta-\alpha)^3$

$=\frac{1}{6}\left(\frac{ウ+\sqrt{D(t)}}{2}-\frac{ウ-\sqrt{D(t)}}{2}\right)^3$

$=\frac{1}{6}\{D(t)\}^{\frac{3}{2}}$

結局 $(6\times S(t))^{\frac{2}{3}}=D(t)$

なので(4)の式より

$t=\frac{2}{3}$ の前後で $D'(t)$ が $-\to+$ より極小．

$\therefore$ 極小値$=D(\frac{2}{3})=\frac{2000}{27}$

$t=2$ の前後で $D'(t)$ が $+\to-$ より極大．

$\therefore$ 極大値$=D(2)=112$