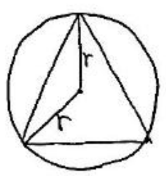
2016 聖マリアンナ

$\boxed{1}$

(1) $\dfrac{9\cdot 8\cdot 7\cdot 6}{4^4}=\underline{\dfrac{112}{243}}$ //

(2)



$\dfrac{1}{2}r\cdot r\underbrace{\sin 120^\circ}_{\frac{\sqrt{3}}{2}}=9\sqrt{3}$

$\Leftrightarrow r^2=36$

$\therefore r=\underline{6}$ //

(3)

(与式) $x=-t$

$=\lim\limits_{t\to\infty}\left(-4t+3+\sqrt{16t^2+9}\right)$

$=\lim\limits_{t\to\infty}\dfrac{16t^2+9-(4t-3)^2}{\sqrt{16t^2+9}+4t-3}$

$=\dfrac{24}{4+4}=\underline{3}$ //

(4)

(分子)

$=\sin(60^\circ-5^\circ)+\sin(60^\circ+5^\circ)$
$+\sin(180^\circ-5^\circ)+\sin(180^\circ+5^\circ)$

$=2\sin 60^\circ\cos 5^\circ+2\sin 180^\circ\cos 5^\circ$

$=\sqrt{3}\cos 5^\circ$

(分母)

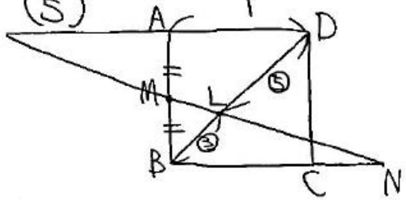
$=\sin(45^\circ+5^\circ)+\cos(45^\circ+5^\circ)$

$=\dfrac{1}{\sqrt{2}}\cos 5^\circ+\dfrac{1}{\sqrt{2}}\sin 5^\circ+\dfrac{1}{\sqrt{2}}\cos 5^\circ-\dfrac{1}{\sqrt{2}}\sin 5^\circ$

$=\sqrt{2}\cos 5^\circ$

$\therefore$ (与式) $=\dfrac{\sqrt{3}}{\sqrt{2}}=\underline{\dfrac{\sqrt{6}}{2}}$ //

(5)



$\triangle ALD$ に余

$AL^2=1^2+\left(\dfrac{5\sqrt{2}}{8}\right)^2-2\cdot 1\cdot\dfrac{5\sqrt{2}}{8}\cos 45^\circ$

$=1+\dfrac{25}{32}-\dfrac{10}{8}$

$=\dfrac{17}{32}$

$\therefore AL=\dfrac{\sqrt{17}}{4\sqrt{2}}=\underline{\dfrac{\sqrt{34}}{8}}$ //

$\boxed{2}$

(1) $c_n=\underline{an}$ //

$a_{k-4}a_{k-1}a_ka_{k+1}$

$=\underline{a^4(k-4)(k-1)k(k+1)}$ //

(2)
(i)

$8\,)\,2016$
$4\,)\,252$
$7\,)\,63$
$\quad 9$

$\underbrace{a^4(k-4)(k-1)k(k+1)}_{\geqq 120}=\underline{2^5\cdot 3^2\cdot 7}$ ……①

$120a^4\leqq 2^5\cdot 3^2\cdot 7$

$\Leftrightarrow\ 5a^4\leqq 4\cdot 3\cdot 7$

これより) $a=1, 2$. しかし $a=2$ は不適.

$(k-4)(k-1)k(k+1)=2016$

探すと $k=8$

$\therefore (a, k)=\underline{(1, 8)}$ //

①の左辺が
2^6 になるため

(ii) $t^3+(t+1)^3=(t+2)^3-2$

$\downarrow\ t+1=T$

$(T-1)^3+T^3=(T+1)^3-2$

$\Leftrightarrow 2T^3-3T^2+3T-1=T^3+3T^2+3T-1$

$\Leftrightarrow\ T^3-6T^2=0$

$\Leftrightarrow T=0, 6$

$\Leftrightarrow\ t=\cancel{-1}, 5$

$\therefore a_{t+1}=a_6=\underline{6}$ //

(iii) $s+5=u$ とおくと

$a_{u-5}^2+a_{u-4}^2+a_{u-3}^2+a_{u-2}^2+a_{u-1}^2+a_u^2$
$=a_{u+1}^2+a_{u+2}^2+a_{u+3}^2+a_{u+4}^2+a_{u+5}^2$

$\Leftrightarrow$

$(u-5)^2+(u-4)^2+(u-3)^2+(u-2)^2+(u-1)^2+u^2$
$=(u+1)^2+(u+2)^2+(u+3)^2+(u+4)^2+(u+5)^2$

$\Leftrightarrow$

$-10u-8u-6u-4u-2u+u^2$
$=2u+4u+6u+8u+10u$

$\Leftrightarrow\ u^2-60u=0$

$\Leftrightarrow\ u=\cancel{0}, 60$

$a_{s+5}=a_{60}=\underline{60}$ //

3

(1)

$f(0)$
$= \int_0^a |-te^t| dt$
$= \int_0^a te^t dt$
$= [te^t - e^t]_0^a$
$= \underline{(a-1)e^a + 1}$ //

$f(a)$
$= \int_0^a |ae^a - te^t| dt$
$= \int_0^a (ae^a - te^t) dt$
$= a^2e^a - f(0)$
$= \underline{(a^2-a+1)e^a - 1}$ //

(2)

$f(x)$
$= \int_0^x (xe^x - te^t) dt$
$\quad + \int_x^a (-xe^x + te^t) dt$
$= [xe^x t - te^t + e^t]_0^x$
$\quad + [-xe^x t + te^t - e^t]_x^a$
$= x^2e^x - xe^x + e^x - 1$
$\quad - axe^x + ae^a - e^a + x^2e^x - xe^x + e^x$
$= 2x^2e^x - (2+a)xe^x + 2e^x$
$\quad + (a-1)e^a - 1$
$= \underline{\{2x^2 - (a+2)x + 2\}e^x + (a-1)e^a - 1}$ //

(3)

$f'(x)$
$= \{2x^2 - (a-2)x - a\}e^x$
$= (2x-a)(x+1)e^x$

$f'(x) = 0 \Leftrightarrow x = \underline{\frac{a}{2}}$ //

(4)

x	$0 \cdots \frac{a}{2} \cdots a$
$f'(x)$	$-\ 0\ +$
$f(x)$	$\searrow\ \ \nearrow$

$\min f(x) = f\left(\frac{a}{2}\right)$
$= \underline{(a-1)e^a - (a-2)e^{\frac{a}{2}} - 1}$ //

次に
$g(a) = f(a) - f(0)$
$= (a^2-2a+2)e^a - 2$ とおく
$g'(a) = a^2e^a > 0$
$g(a)$は単調増加. $g(0)=0$より
$g(a) \geq 0$
$\therefore \max f(x) = f(a) = \underline{(a^2-a+1)e^a - 1}$ //

4

(1)

$a^3 = p(p^2abc - b^3 - c^3)$
よりa^3はpの倍数なので
aもpの倍数.

(2) (1)より
$a = kp \ (k \in \mathbb{Z})$
とおけるので
$pb^3 = -a^3 - p^2c^3 + p^3abc$
$\Leftrightarrow b^3 = p(-k^3p - c^3 - pabc)$
(1)と同様に bはpの倍数.
$b = lp \ (l \in \mathbb{Z})$
とおくと
$p^2c^3 = -k^3p^3 - l^3p^4 + p^3abc$
$\Leftrightarrow c^3 = p(-k^3 - l^3p + abc)$
同様にcもpの倍数.

(3) $c = mp \ (m \in \mathbb{Z})$
とおくと
$k^3p^3 + l^3p^4 + m^3p^5 - klmp^6 = 0$
$\Leftrightarrow k^3 + l^3p + m^3p^2 - klmp^3 = 0$
同様にk, l, mがpの倍数となる. これよりa, b, cはpで無限に割れる. pで無限に割れるのは0だけなので
$a = b = c = 0$

(4)

$x = \frac{x_2}{x_1}, \ y = \frac{y_2}{y_1}, \ z = \frac{z_2}{z_1}$
$(x_1, x_2, y_1, y_2, z_1, z_2 \in \mathbb{Z}$
$x_1y_1z_1 \neq 0)$
とおくと
$\left(\frac{x_2}{x_1}\right)^3 + p\left(\frac{y_2}{y_1}\right)^3 + p^2\left(\frac{z_2}{z_1}\right)^3 - p^3\frac{x_2y_2z_2}{x_1y_1z_1} = 0$
$\Leftrightarrow (y_1z_1x_2)^3 + p(x_1z_1y_2)^3 + p^2(x_1y_1z_2)^3$
$\quad - p^3(x_1^2y_1^2z_1^2x_2y_2z_2) = 0$
(3)より
$y_1z_1x_2 = x_1z_1y_2 = x_1y_1z_2 = 0$
$\Leftrightarrow x_2 = y_2 = z_2 = 0$
$\therefore x = y = z = 0$