2015 聖マリアンナ

$\boxed{1}$

[1]

$\cos 3\theta$
$= 4\cos^3\theta - 3\cos\theta$

$\Leftrightarrow \cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$ より

(与式)

$= 8\left\{1+\frac{1}{4}\cos\frac{\pi}{6}+\frac{3}{4}\cos\frac{\pi}{18}\right\}\frac{1}{2}$

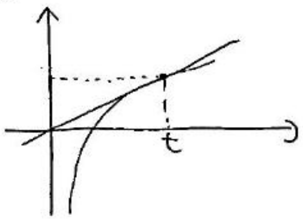
$-\frac{3}{2}\left(1+\cos\frac{\pi}{18}\right)2$

$= 4\left(1+\frac{\sqrt{3}}{8}+\frac{3}{4}\cos\frac{\pi}{18}\right)$

$-3\left(1+\cos\frac{\pi}{18}\right)$

$= \underline{1+\frac{\sqrt{3}}{2}}$ //

[2]



$\begin{cases} \text{傾き一致} & \frac{1}{t} = b \\ y\text{座標一致} & \log t = bt \end{cases} \rightarrow$ 接するのは $b = \frac{1}{e}$

$\therefore \underline{0 < b < \frac{1}{e}}$ //

[3]

$C_n = 2n-1$

$S_n = \sum_{k=1}^{n}(2k-1)$

$= n^2$

$U_n = e^{\log S_n}$

$= S_n$

$= n^2$

$U_1 + \cdots + U_{24}$

$= \sum_{k=1}^{24} k^2$

$= \frac{1}{6}\cdot 24\cdot 25\cdot 49 = \underline{4900}$ //

[4]

(与式)

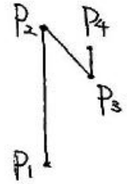
$= \left[\log(2e^x+3)\right]_0^{\log 3}$

$= \log 9 - \log 5$

$= \underline{2\log 3 - \log 5}$ //

$\boxed{2}$

[1]



$P_3\left(1+\frac{1}{3\sqrt{2}},\ 2-\frac{1}{3\sqrt{2}}\right)$

$P_4\left(1+\frac{1}{3\sqrt{2}},\ 2-\frac{1}{3\sqrt{2}}+\frac{1}{9}\right)$.

$\therefore \overrightarrow{P_3P_4} = \underline{\begin{pmatrix} 0 \\ \frac{1}{9} \end{pmatrix}}$ //

[2]

以後同様に

$\overrightarrow{P_{2k+1}P_{2k+2}} = \frac{1}{9}\overrightarrow{P_{2k-1}P_{2k}}$.

$\overrightarrow{P_1P_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ より

$\therefore \overrightarrow{P_{2k-1}P_{2k}} = \left(\frac{1}{9}\right)^{k-1}\overrightarrow{P_1P_2}$

$= \underline{\begin{pmatrix} 0 \\ \left(\frac{1}{9}\right)^{k-1} \end{pmatrix}}$ //

[3]

同様に

$\overrightarrow{P_{2k+2}P_{2k+3}} = \frac{1}{9}\overrightarrow{P_{2k}P_{2k+1}}$.

$\overrightarrow{P_2P_3} = \begin{pmatrix} \frac{1}{3\sqrt{2}} \\ -\frac{1}{3\sqrt{2}} \end{pmatrix}$

$\therefore \overrightarrow{P_{2k}P_{2k+1}} = \left(\frac{1}{9}\right)^{k-1}\overrightarrow{P_2P_3}$

$= \begin{pmatrix} \frac{\sqrt{2}}{6}\left(\frac{1}{9}\right)^{k-1} \\ -\frac{\sqrt{2}}{6}\left(\frac{1}{9}\right)^{k-1} \end{pmatrix}$

[4]

$\overrightarrow{P_{2k-1}P_{2k+1}}$

$= \overrightarrow{P_{2k-1}P_{2k}} + \overrightarrow{P_{2k}P_{2k+1}}$

$= \begin{pmatrix} \frac{\sqrt{2}}{6}\left(\frac{1}{9}\right)^{k-1} \\ \left(1-\frac{\sqrt{2}}{6}\right)\left(\frac{1}{9}\right)^{k-1} \end{pmatrix}$

$\begin{pmatrix} X \\ Y \end{pmatrix} = \overrightarrow{OP_1} + \lim_{n\to\infty}\sum_{k=1}^{n}\overrightarrow{P_{2k-1}P_{2k+1}}$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \dfrac{\frac{\sqrt{2}}{6}}{1-\frac{1}{9}} \\ \dfrac{1-\frac{\sqrt{2}}{6}}{1-\frac{1}{9}} \end{pmatrix}$

$= \underline{\begin{pmatrix} 1+\frac{3\sqrt{2}}{16} \\ \frac{17}{8}-\frac{3\sqrt{2}}{16} \end{pmatrix}}$ //

$\boxed{3}$

[1] $\dfrac{BC}{2} = AB\sin\theta$

$\therefore L = AB+BC+CA$

$= AB + 2AB\sin\theta + AB$

$= 2AB(1+\sin\theta)$

$\Leftrightarrow AB = \dfrac{L}{2(1+\sin\theta)}$

また

$\triangle ABC = \dfrac{a}{2}(AB+BC+CA)$

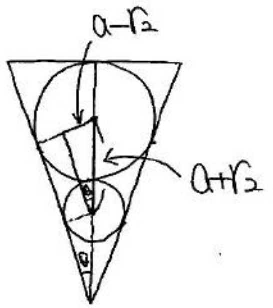
$\Rightarrow \dfrac{1}{2}AB^2\sin 2\theta = \dfrac{aL}{2}$

$\Rightarrow \dfrac{L^2}{4(1+\sin\theta)^2}\sin 2\theta = aL$

$\Leftrightarrow L\sin 2\theta = 4a(1+\sin\theta)^2$

$\therefore L = \underline{\dfrac{2a(1+\sin\theta)^2}{\sin\theta\cos\theta}}$ //

[2]



円O_nの半径をr_nとおく.

図より

$\sin\theta = \dfrac{a-r_2}{a+r_2}$

$\Leftrightarrow (1+\sin\theta)r_2 = a - a\sin\theta$

$\Leftrightarrow r_2 = \dfrac{1-\sin\theta}{1+\sin\theta}a$

以後同様に

$r_n = a\left(\dfrac{1-\sin\theta}{1+\sin\theta}\right)^{n-1}$

よって

W

$= \displaystyle\sum_{n=1}^{\infty} 2r_n\pi$

$= 2a\pi\displaystyle\sum_{n=1}^{\infty}\left(\dfrac{1-\sin\theta}{1+\sin\theta}\right)^{n-1}$

$= 2a\pi\cdot\dfrac{1}{1-\dfrac{1-\sin\theta}{1+\sin\theta}}$

$= \underline{\dfrac{\pi a(1+\sin\theta)}{\sin\theta}}$ //

[3]

$L = W$

$\Leftrightarrow \dfrac{2(1+\sin\theta)}{\cos\theta} = \pi$

$\Leftrightarrow 2+2\sin\theta = \pi\cos\theta$

$\Leftrightarrow \sin\theta = \dfrac{\pi}{2}\cos\theta - 1$

$\sin^2\theta + \cos^2\theta = 1$ より

$\left(\dfrac{\pi}{2}\cos\theta - 1\right)^2 + \cos^2\theta = 1$

$\Leftrightarrow \underbrace{\cos\theta}_{\neq 0}\left\{\left(\dfrac{\pi^2}{4}+1\right)\cos\theta - \pi\right\} = 0$

$\therefore \cos\theta = \dfrac{\pi}{\frac{\pi^2}{4}+1}$

$= \underline{\dfrac{4\pi}{\pi^2+4}}$ //

$\therefore \sin\theta = \dfrac{2\pi^2}{\pi^2+4} - 1 = \underline{\dfrac{\pi^2-4}{\pi^2+4}}$ //

$\boxed{4}$

[1]

(1) $\displaystyle\int_0^a te^{-t}dt$

$= \left[-te^{-t} - e^{-t}\right]_0^a$

$= -ae^{-a} - e^{-a} + 1$

$= \underline{-(a+1)e^{-a} + 1}$ //

(2) $\displaystyle\int_0^a t^2e^{-t}dt$

$= \left[-t^2e^{-t} - 2te^{-t} - 2e^{-t}\right]_0^a$

微分 $\left\{ \boxed{t^2e^{-t} - 2te^{-t}} \boxed{+2te^{-t} - 2e^{-t}} \boxed{+2e^{-t}} \right.$

$= -a^2e^{-a} - 2ae^{-a} - 2e^{-a} + 2$

$= \underline{-(a^2+2a+2)e^{-a} + 2}$ //

[2]

$f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}}e^{-\sqrt{x}}$

$+ (\sqrt{x}-1)\left(-\dfrac{1}{2}x^{-\frac{1}{2}}\right)e^{-\sqrt{x}}$

$= \left(x^{-\frac{1}{2}} - \dfrac{1}{2}\right)e^{-\sqrt{x}}$

$= \dfrac{2-\sqrt{x}}{2\sqrt{x}}e^{-\sqrt{x}}$

$x = \underline{4}$ // で極大値 $e^{-2} = \underline{\dfrac{1}{e^2}}$ //

$(t, f(t))$での接線

$y = \dfrac{2-\sqrt{t}}{2t}e^{-\sqrt{t}}(x-t) + (\sqrt{t}-1)e^{-\sqrt{t}}$

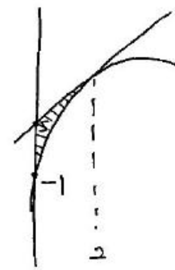
$\downarrow$ 原点通る

$0 = \dfrac{2-\sqrt{t}}{2}(-\sqrt{t})e^{-\sqrt{t}} + (\sqrt{t}-1)e^{-\sqrt{t}}$

$\Leftrightarrow 0 = t - 2\sqrt{t} + 2\sqrt{t} - 2$

$\therefore t = \underline{2}$ //

(傾き) $= \underline{\dfrac{\sqrt{2}-1}{2}e^{-\sqrt{2}}}$ //



S

$= \displaystyle\int_0^2 \left\{\dfrac{\sqrt{2}-1}{2}e^{-\sqrt{2}}x - (\sqrt{x}-1)e^{-\sqrt{x}}\right\}dx$

$= \cdots$

$= \underline{(3\sqrt{2}+5)e^{-\sqrt{2}} - 2}$ //